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Criteria for Aims in Mathematics

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TEACHING aims have always held an important place in the literature of secondary school mathematics. In any given period they clearly reveal the current psychological theories, social problems, and schools of educational thought. At the turn of the century, for example, we discern the influence of faculty psychology and the theory of mental discipline. In the analytical 'twenties the point of view of the job analysis was reflected in detailed and cumbersome lists of aims that broke down of their own weight. Since the early 'thirties our society has been threatened by economic forces from within and totalitarian forces from without. Accordingly we find that educational aims are preoccupied by definition of an increased responsibility of education in a free society. This is all to the good if we can close the gap between the expressed aims and the implications for classroom activity.

The aims that are set up for a given topic, if they are to serve their purposes effectively, must be sufficiently complete and definite to orient the classroom activities, to direct the selection of materials and resources, and to serve as the basis for evaluation. Rarely does a teacher need to ask where to find aims. They exist in confusing and profuse abundance on all sides. Books, periodicals, courses of study, institutes, other teachers, pupils, and the lay public are continually proposing more aims than one can possibly use. The problem is which of these possible aims to select. In other words, what criteria can be established by means of which a list of aims may be appraised?

In proposing such criteria, major attention in what follows will be devoted to mathematics. These criteria are general in application, however, and are equally valid in any other field. The proposed criteria are these:

***Validity.** Can the aims be justified on the basis of (1) social needs, (2) personal needs of the pupils, and (3) feasibility as determined by the possibility of attaining them?*

***Comprehensiveness and Selectivity.** Do the aims include the outcomes of major importance and represent the unique contributions of the field, and differentiate the more important from the less important?*

***Suitability of Form.** Are the aims sufficiently defined to suggest the content, experiences, and evaluation practices that follow from them?*

Each of these three criteria requires some attention and illustration. They will be considered in order as listed.

VALIDATION—1. SOCIAL

The question here is whether the aim expresses an important social need. It is a recognized fact that society is becoming more and more quantitative. Greater demands are placed on the individual in the field of quantitative thinking, quanti-

tative communication, and quantitative problem-solving. Priority must be given to the most important of these needs. As Breslich[1] has pointed out, any subject-matter area must find its aims among the general aims of education, and its value must be judged by the contribution it makes to those aims.

Except for aims which have a strictly subject-matter bearing, or are based on remote and problematical college requirements, the aims commonly listed for the field of mathematics usually meet this social requirement. An exception lies in a group of aims somewhat carelessly stated which suggest the need for extensive research in order that their social validation may rest on solid ground.

Thus, "Speed and accuracy of computation (in a given process)"; reveals, on examination a number of loose ends that suggest needed research. How much speed is desirable for this process? After the attainment of a certain degree of speed in any process, the law of diminishing returns takes effect and more and more time is required to increase the speed by any appreciable amount. How much accuracy is necessary? With human beings, one hundred per cent accuracy is not attainable. We need information as to the optimum level of accuracy for any process, and the point at which the tendency to check answers is permissible as a substitute for perfect accuracy.

The above illustration indicates the inadequacy of our customary arm-chair and round-table technique of determining social needs. We need definite information on the competences required for adult activity. Not only do we need a tabulation of the types of adult activities for which mathematical competence is a prerequisite. We need also a set of minimum requirements for these activities, to serve as a basis for establishing standards for speed, accuracy, information, and methodology.

[1] E. R. Breslich, *The Administration of Mathematics In Secondary Schools.* University of Chicago Press, 1933. Pp. 151–2.

There is, of course, a considerable amount of information of this sort available from research already carried out. However, a considerable amount that found its way into bibliogoraphies is of little or no value because of inadequacy of the research or the carelessness with which inferences have been drawn. It is necessary that this data be collected, and then supplemented by further research.

Validation—2. Personal Needs

We may properly assume that if there is a social need for a given mathematical competence, then the personal need follows as a matter of course. We are here concerned, however, with the question of the psychology of motivation in particular, and the psychology of learning in general. This question is briefly: Can the pupil be led to appreciate the personal significance of the aim?

In general, to meet this criterion the pupil is learning to *do* something important to him, whether it is to appreciate geometric forms in his environment, or to measure inaccessible heights.

It is true that the following aim:

"To acquire ability to divide a four place number by a two place number with a (defined) speed and accuracy";

may become significant to the pupil under the following conditions:

1. The pupil recognizes the significance of the skill in order to carry out a computation which he desires to make.
2. Through testing, the pupil discovers his inadequacy in the skill.
3. The pupil recognizes the cause of his inadequacy.
4. The pupil has confidence in his ability through suitable remedial work to correct his inadequacy.

Other requirements for effective drill procedures are familiar to the reader. It is important to note, however, that they carry the implication that the pupil has seen the mathematical skill as a prerequisite to some ability important to him. There is a considerable body of reliable information in the field of pupil interest, motivation, and the psychology of learn-

ing. Much more of the same sort, however, is necessary before the validation of objectives on the basis of personal needs can be anything more than tentative.

Validation—3. Feasibility

The question here is whether it is possible to achieve the proposed outcome through the teaching of mathematics. If so, a further question must be. Can it be achieved with reasonable expenditure of time and effort?

The aims of any given period, as we have noted, reflect the psychological theories of the period. Aims from previous periods, based on abandoned psychological assumptions tend to persist. We still have, for example, aims based on faculty psychology and mental discipline.

Consider, for example, development of the trait of *honesty* as an outcome in the field of mathematics. As proposed, it appears plausible that from dealing with eternal truth in his mathematical experiences the pupil must develop a respect and appreciation for it, and hence become more honest. Some doubt is thrown on the existence of such a general trait by certain studies. Mathematical historians, moreover, have not shown that mathematicians as a group are more honest than comparable groups in other fields. Hence it would be unsound to evaluate the effectiveness of mathematics teaching on the basis of this outcome until more is known as to the existence and origin of such traits.

The Comprehensive and Selective Characteristic

Any set of aims that has been brought together in a hap hazard way will almost inevitably neglect some aims that are of slight importance. It is true that we have not the time nor resources to realize all the outcomes possible from the teaching of mathematics. It is of the utmost importance, therefore, that we examine each topic to consider all the important outcomes to which it can contribute, and to differentiate those that are important from those that are relatively subordinate.

If we go beyond those that are mere verbalisms, aims commonly listed reveal an amazing lack of imagination and appreciation of the possibilities of the field of mathematics. It is always useful to raise the question, how do people really use this aspect of mathematics? Here we must take into account a variety of points of view regarding the field.

In the first place, any one of the great subject-matter fields has two aspects, each of which is essential. One is the methodology which the field provides in attacking a problem; the second is the body of information which has been accumulated through the ages and which is useful in implementing the methodology. It is obvious that the methodology of mathematics is the quantitative approach to a problem situation. This is true whether we are merely thinking through a situation, communicating ideas about a situation, or formally attacking a problem. The aims which we set up for the field, then, must reflect these various aspects of the field: the methodology and the informational aspects; and the use of both methodology and information in thinking, communicating, and problem-solving.

It is important also that we do not overlook either the utilitarian or appreciation outcomes of mathematics. Great attention has been given in the literature to appreciation of the power of mathematics; appreciation of geometric aspects of our environment; appreciation of the beauty and effectiveness of the mathematical method; and the like. It is relatively rare, however, to see these aims defined in such a way as to direct the classroom teacher to procedures which are appropriate to achieving these aims, and make possible an evaluation of the effectiveness with which they are achieved.

The utilitarian aims are those which express the ways in which mathematics can make the individual more effective personally and socially. They are the ones

which are the basis of that somewhat annoying remark "After all, mathematics is a tool subject." It is true that mathematics is a tool subject, as in any field which makes it possible for the individual to be able to do something. So long as the consummatory aims remains merely verbalism, we must admit that mathematics is "merely" a tool subject. True, "merely" to be able to deal effectively with the quantitative aspects of the social and physical environment would be quite an achievement for anyone in this day and age. But we should not overlook the broad opportunities for developing interest, attitudes, and appreciations in the field which would make the world more interesting to the pupil and the pupil more interesting to the world.

Turning to the selective characteristic required of any list of aims, it is important that we should discover the presence of proposed outcomes that can be achieved only at the expense of others that are more important. In view of the increasingly precise and quantitative nature of our social, industrial, and technical environment, it is difficult to challenge any possible outcome from the teaching of mathematics when viewed solely by itself. It is only when we realize that accepting any one aim implies the rejection of others that we appreciate the necessity of taking into account the law of diminishing returns and the justification of comparative values in any list of aims. To take an extreme, but real, example:

> *Ability to multiply a three-place number by a three-place number mentally.*

While it can be achieved, its value must be compared to that of other outcomes that must be neglected in so doing.

Suitability of Form

Having selected the aims, we must next proceed to express them in a form that will be useful both as a basis for teaching and for evaluation. For this purpose, it must meet the following criteria:

1. Objectivity: Will two competent observers agree as to whether a given pupil has achieved the aim?
2. Definiteness: Does it suggest the kind of learning activity that should be undertaken?
3. Differentiating Value: Does it suggest the kind of evidence that is suitable for evaluating the outcomes of instruction that are implied?

To take for an example from the report of the Commission on Post War Mathematics, we may quote the following expression of an aim:

"Does he have a clear understanding of ratio?" For the sake of argument, we may agree that this aim is valid socially, personally, and pedagogically, Is it expressed in such a way as to be effective as a basis in teaching and evaluation?

First, as to objectivity, What do we mean by *understanding?* Is the pupil to understand in the passive way an idea in which ratio is expressed? Or, is he to understand it to a degree where he can establish the ratio between quantities? Furthermore, what is meant by a clear understanding? Certainly, a one hundred percent performance on every level is not implied. It is a fair question, however, to inquire what level of performance *is* implied?

As to definiteness, what applications of ratio are to be included here? It would be unfair to ask the Commission to list all of the applications of ratio that could be contemplated under this objective. However, an illustrative list to reveal the kind of thinking that went on in the Commission meeting would be very enlightening. Moreover, there are many ways of expressing ratios. Which of these ways of expression did the Commission have in mind? All of these questions are very pertinent to the teacher who takes the Commission report seriously, and wishes to supplement it in her class work.

Turning to the differentiating value, which would serve as a basis for evaluation, what would the pupil be able to do who had achieved this aim as contrasted to the pupil who had not achieved the aim?

As an example of a similar aim, ex-

pressed in such a way as to meet the criteria of objectivity, definiteness, and differentiating value, we may suggest the folowing:

Given the angle of elevation at a given distance from a building, the pupil should be able to use the tangent ratio to determine the height of the building.

A list of such illustrative definitions would give meaning to the aim "To have a clear understanding of ratio." Without such a list of illustrative abilities, the aim is relatively meaningless.

In recognition of the need to express aims in a suitable form, the operational definition is being utilized to an increasing extent. This method of expressing aims in term of desired behavior has many advantages—it is simple, and it avoids verbalism, as well as the pitfalls of expressing outcomes in terms of traits. To derive an operational definition for a given aim, several key questions must be asked:

1. Can I recognize this competence in a pupil?
2. Can I differentiate a pupil who has the competence from one who does not?
3. What kinds of behavior will distinguish the pupil who has the competence from one who does not have it?

To take an illustration: Most teachers will agree that an important aim in mathematics is the development on the part of the pupil of an interest in mathematics. We may agree that this aim is valid socially, personally, and pedagogically. In its form as stated however, it does not meet the criterion of form. We must define certain typical behavior of a pupil who is interested in mathematics, as contrasted to one who is not.

Let us take the key questions; Can we recognize interest in a pupil? Hardly anyone would presume to say we could not. How can we differentiate a pupil who has interest from one who has not? Here we could list a series of differentiating behaviors such as the following:

1. Is attentive to class discussion.
2. Asks pertinent questions.
3. Volunteers pertinent suggestions to the discussion.
4. Brings in contributions from outside the class.
5. Voluntarily does more than the required amount of assigned work, etc.

No experienced teacher would assume that any one of these operational definitions establishes the fact of interest in a pupil any more than one swallow makes a summer. Neither must we insist that all be present in a given pupil. As these evidences accumulate, however, we have an objective indication of the degree of interest, and the varying amounts and manifestations of interest in various members of the class.

Application of these criteria to the so-called "intangible" outcomes such as appreciations and attitudes would have a particularly important effect in clarifying our thinking as to the kinds of behavior we are seeking, the situations in which such behavior should be manifested, and the classroom experiences that might be expected to produce such behavior. For all types of outcomes, however, the utilization of these criteria should result in increasing their effectiveness for orientation of classroom activities, direction of selecting materials and resources, and serving as the first step in an evaluation program.

A. H. Albaugh read this - pass on ye lit... Come...

Mathematics in the History of Civilization*

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THE discussion of the subject at hand would gain in clarity if I could ask the members of my invisible audience whether they can remember themselves far enough back in their childhood, when they were not able to count, that is when they could not recite the series of words one, two, three, four, and so on. Radio being what it is, the question must remain unanswered. Fortunately, I know what the answer would be, if it could be given. It is just like asking somebody whether he can recall the time when he could not walk or when he could not talk.

The same situation prevails with regard to the human race as a whole. We know quite definitely that there was a time when the notion of number was totally alien to mankind. Who was the genius who first asked the momentous question: "How many?" We will never know. At a certain stage of social development the need arises to determine how many objects constitute a given collection. The answer to the question becomes a social necessity. Contributions towards finding that answer are made by many individuals confronted with the same need, and the notion of number slowly emerges.

How slow and painful a process of creation this was may be judged from the fact that there are human tribes whose languages have no words for numbers greater than four, and even no greater than two. Beyond that any group consists of "many" objects.

Our numbers are applied to any kind of objects in the same way, without discrimination, they have a kind of "impersonality." That is not the attitude of the primitive man. With him the number applied to a group is modified in accordance with the nature of the group. The number characterizes the group in the same way as an adjective applied to a noun describes the object to which it is applied. The English language has preserved some traces of that attitude. A group of cattle is a *herd*, while a group of birds is a *flock*; a group of wolves is a *pack*, while a group of fish form a *school*. It would be shocking indeed to speak of a school of cows. Other languages offer much more striking proofs of such an attitude towards numbers in their relation towards the objects they are applied to. Thus in English we use the singular grammatical form when one object is involved, and we use the plural grammatical form for any number of objects larger than one. Some of the languages of the Western world, in their earlier stages of development, had a special grammatical form, a dual form, when two objects were spoken of. Some languages even had separate grammatical forms when reference was made to three objects, and still another form for four objects. An instructive example of the way the form of the same number may be modified to fit the group to which it is applied is furnished by the Polish language in its use of the number two. In that language a different form of "two" is used when applied to two men, to two women, to a man and a woman, and to inanimate objects or animals. These forms are, respectively: *dwaj, dwie, dwoje, dwa.*

The process of accumulating enough words to answer the question: how many? to satisfy the growing needs was slow and laborious. Man derived a great deal of help from the natural set of counters he always carries with him—his fingers. Of the many examples that could be cited to illustrate the use of fingers as counters let

* The two parts of this article were delivered over station WNAD as the opening broadcasts of a series under the general title "Mathematics in Human Affairs" arranged by the Department of Mathematics of the University of Oklahoma.

me quote a report of Father Gilij who described the arithmetic of the Indian tribe of the Tamanacas, on the Orinoco river.

The Tamanacas have words for the first four numbers. When they come to five they express it by a phrase which literally means "a whole hand"; the phrase for the number six means literally "one on the other hand," and similarly for seven, eight, and nine. When they come to ten, they use the phrase "both hands." To say eleven they stretch out both hands, and adding a foot, they say "one on the foot," and so on, up to 15, which is "a whole foot." The number 16 is "one on the other foot." For twenty they say "one Indian," and 21 is expressed by saying "one on the hands of the other Indian"; forty, sixty, . . . is "two indians," "three Indians," and so on.

When the question: how many? has once been raised, mere counting becomes insufficient. Further steps in civilization bring about the need of computation. The strongest single factor that stimulated the development of methods of computation was trade. According to the mythology of the ancient Egyptians, arithmetic was invented by their god of commerce. As with counting, the beginnings of reckoning were slow and laborious, awkward and painful. A trader in tropical South Africa during the last century has this to say about the members of the Dammara tribe. "When bartering is going on, each sheep must be paid for separately. Thus, suppose two sticks of tobacco to be the rate of exchange for one sheep; it would sorely puzzle a Dammara to take two sheep and give him four sticks. I have done so, and seen a man put two of the sticks apart and take a sight over them at one of the sheep he was about to sell. Having satisfied himself that *that* one was honestly paid for, and finding to his surprise that exactly two sticks remained in his hand to settle the account for the other sheep, he would be afflicted with doubt; the transaction seemed to come out too "pat" to be correct, and he would refer back to the first couple of sticks; and then his mind got hazy and confused, and he wandered from one sheep to the other, and he broke off the transaction, until two sticks were put in his hand, and one sheep driven away, and then two other sticks given him and the second sheep driven away." It would seem that at least to this representative of humanity it was not obvious that two times two makes four.

The story illustrates the blundering beginnings of the art of reckoning. To relate the evolution of this art from its humble beginnings to the heights of power and perfection it has achieved in modern times, and how this art has followed and served the ever growing needs of mankind is to tell one of the most exciting sagas in the history of civilization. Only a mere outline can be attempted here.

Various human activities, and in particular commerce, require the keeping of some numerical records. Some kind of marks had to be invented for the purpose. The devices used through the ages were many and various. Among them were knots tied in a rope and notches cut in sticks. It may surprise some of my readers that such sticks, called *tallies*, were used as a method of bookkeeping by the Bank of England way into the nineteenth century.

The first written symbols for numbers were, naturally, sticks: *one stick*, *two sticks*, *three sticks*, and so on, to represent "one" "two," "three" etc. This worked fairly well as long as the numbers to be represented were small. For larger numbers the sticks occupy too much space, it becomes difficult to count them, and it takes too much time. The sticks had to be condensed into groups, thus representing larger units, and these new units in turn had to be condensed into larger units and thus a hierarchy of units had to be formed. The Greeks and the Hebrews used the letters of their alphabets as numerals. The Babylonians had special numerical symbols.

All these symbols or marks for numbers had one feature in common—they did not lend themselves to arithmetical computations. The art of reckoning had to be carried out with the help of different devices, the chief among them being the counting frame, or the *abacus*. This instrument most often consisted of a rectangular frame with bars parallel to one side. The operations were performed on the beads or counters strung on these bars. This instrument was widespread both in Asia and in Europe. When the Europeans arrived in America they found that the abacus was in use both in Mexico and in Peru.

The method of writing numbers and computing with them that we use now had its origin in India. The most original feature of that system, namely the Zero, the symbol for nothing, was known in Babylon and became common in India during the early centuries of the Christian era. Due to its flexibility and simplicity it gradually forced out the abacus. This system was brought to Europe by the Arabic and Jewish merchants during the twelfth century. The first printing presses set up in Europe, in the middle of the fifteenth century, rolled off a considerable number of commercial arithmetics. Two centuries later the abacus in Western Europe was little more than a relic of the past.

The very heavy demands that modern life in its various phases makes upon computation seem to be turning the tide against paper and pencil reckoning. We are about to enthrone the abacus back again, in a much improved form, to be sure, but nevertheless in the form of an instrument. In fact, we are using a considerable number of them, like the slide rule, the cash registers, the various electrically operated computers, to say nothing of the computing machines which operate on a much higher level, like those which give the solutions of differential equations. Such is the devious and puzzling road of human progress.

"How many?" This question is the origin of arithmetic and is responsible for much of its progress. But this question cannot claim all the credit. It must share the credit with another, a later arrival on the scene of civilization, but which is even more far reaching. This question is: "how much?" How much does this rock weigh? How much water is there in this barrel? How much time has passed between two given events? How long is the road from town A to town B? etc. The answers to these questions are numbers, like the answer to the question; "how many?" There is, however, a vast difference between the numbers which answer the two kinds of questions.

The answer to the question: "how many?" is obtained by counting discrete objects, like sheep, trees, stars, warriors, etc. Each of the objects counted is entirely separate from the others. These objects can be "stood up and be counted." Something vastly different is involved in the question: How much does this rock weigh? The answer can only be given by comparing the weight of the given rock to the weight of another rock, or to the weight of some other object taken for the unit of weight, say a pound or a ton. Obviously this is a much more involved process and implies a much more advanced social and intellectual level than the answer to the question: how many?

The question: "how many?" is always answered by an integer. Not so the question: "how much?" Given 17 trees, is it possible to plant them in five rows so that each row would have the same number of trees? The answer is: "No," and this is the end of the story. But given seventeen pounds of salt in a container, it is possible to distribute this salt into five containers so that each of them will hold the same amount of salt. But the question: "how many pounds of salt does each container hold?" cannot be answered by an integer. Thus, the question: "how much?" is responsible for the invention of fractions. It is also responsible for the introduction of irrational numbers. But

about that we may have to say something later on.

The question: "how much?" that is, the introduction of measurements, has involved us in another kind of difficulty which did not bother us in connection with the question: "how many?" We can ascertain that the group at the picnic consisted of forty boys. But when we say that this table is forty inches long, we can only mean that it is closer to forty inches than it is either to 39 or 41 inches. We may, of course, use more precise instruments of measurement. That may narrow down the doubtful area, but it will not remove it. Results of measurements are necessarily only approximations. The degree of approximation to which we carry out these measurements depends upon the use we are to make of these measured things.

The herdsman is much concerned with the question: "how many?" The shepherd, in addition, is also interested in the question: "how much?" after he is through sheering his flock. When a human tribe turns to agriculture, the question: "how much?" imposes itself with increased insistence. Agriculture requires some methods of measuring land, of measuring the size of the crop, that is measuring areas and volumes as our school books call it. Furthermore, the agricultural stage of society implies already a considerable degree of social organization, and the tax collector appears on the scene. This official is vitally interested in the size of the crop. He also has to have some numerical records of the amount of taxes collected and of the amount of taxes due. Now you may not like the tax collector. Few people waste too much love on this maligned official. It is nevertheless quite obvious that no organized society is possible without the collection of taxes, that is without contributions from the individual members of that society towards the necessary enterprises that are of benefit to the members of the entire community. And such collections cannot be made in any orderly fashion, unless answers can be given to the two questions: "how much?" and "how many?"

The cultivation of the land faced the human race with problems of geometry. Egypt with its peculiar dependence upon the flood waters of the river Nile was confronted with extra difficulties of a geometrical nature. That is the reason why geometry found such fertile soil in the valley of the Nile.

Much geometry had to be discovered in order to construct human habitations. When civilization progresses beyond the cave dwelling stage, shelter becomes a problem of the first magnitude. The construction of dwellings involves in the first place the knowledge of the vertical direction, as given by the plumb line. It was observed very early that the plumb line or a pole having the same direction as the plumb line makes equal angles with all the lines passing through its foot and drawn on level ground. We have thus what we call a right angle, as well as the famous theorem of our text books that all right angles are equal.

However important the answers to the question: "How much?" may have been in the connections we just considered, the most important answer to this question is the one connected with the measuring of time. With the most rudimentary attempts at agricultural activity comes the realization that success is dependent upon the seasons; this dependence is even exaggerated. We still worry about the phases of the moon when we want to plant our potatoes.

Various tribes on the surface of the globe noticed that the shortest shadow cast by a vertical pole during the day always has the same direction. This is the north and south direction. The sun at that time occupies the highest point in the sky. It is essential to have a way of marking this direction. Here is how it can be done.

A circle is drawn on the ground having for center the foot of the pole used in the observation. The two positions of the

shadow are marked, the tips of which just fall on the circumference. The north-south line sought is the line mid-way between the two lines marked, and that north-south line was found by many human tribes by bisecting this angle, and was done by the methods still in use in our text-books.

Measurements connected with the sun, the moon, and the stars in general cannot be made directly. Some round-about method must be used. Neither could the size of the earth be determined directly. On the elementary level such artifices are based on geometry and trigonometry. Two centuries B. C. Eratosthenes, the librarian of the famous Alexandrian Library, succeeded by the use of such methods to determine the length of the diameter of the earth with a surprising degree of accuracy. He thus made his contemporaries realize that the world they knew was only a very small part of the surface of the earth.

These very sketchy indications may have given you an idea of the role mathematics played in the development of mankind from the earliest times up to the time of the great civilizations of antiquity.

Part II

The Renaissance was the age of the revival of secular learning in Europe. It was also the age of the great voyages and of the discovery of America, the age of gun-powder and of mechanical clocks.

The new interest in seafaring has raised many pressing problems that had to be solved. The most obvious one was the need for a way of determining the position of a ship on the high seas, that is the need of determining the longitude and the latitude of the ship at any time. This involved a great deal of laborious computation. The invention of logarithms reduced this labor to a fraction of the work it used to require. This accounts for the great success that the invention of logarithms enjoyed, as soon as it became available.

The process of finding the longitude requires an accurate clock which could be relied upon. I pointed out in Part I, above the important role the need of determining the seasons played in the history of civilization, and the mathematical problems that had to be solved in this connection. The navigation of the Renaissance required the measuring of time with great precision. It was a question not of seasons and days but of minutes and seconds. The instrument that made such an accuracy possible was the mechanical clock moved by a pendulum, then by springs. This moving mechanism raised many problems of a mathematical nature that the available mathematical resources were insufficient to cope with. New mathematical methods were needed.

New mathematical problems were also raised by the cannon. It may be observed, in passing, that a cannon was just as much a necessary piece of equipment of a ship starting out on a long voyage towards unexplored shores as was a map, or a clock.

A gunner is frequently in need of determining the distance to some inaccessible objects. The information can be obtained by indirect measurements. This is a problem that was met with much earlier in the history of civilization and was solved in different ways. The cannon has stimulated further development in this connection, thus contributing to the progress of trigonometry.

But artillery presented problems of a new type. The cannon ball was an object which moved with a speed that was unprecedented in the experience of man. Motion took on a new significance and called for mathematical treatment and study. It required the study of the path that the projectile describes in the air, the distance it travels, the height it reaches at any given distance from the starting point, and so on. In short it required what we now call a graph.

The computation of the longitude of a ship at sea is based on astronomical observations and computations made in ad-

vance and published for that purpose. The greater the accuracy of these data, the more correctly can the position of the ship be determined. Thus navigation made necessary a more accurate knowledge of the motion of heavenly bodies.

The mathematics that the Renaissance inherited from preceding periods was inadequate for the study of motion. The new mathematical tools that were invented for the purpose of answering the new questions raised were: (1) Analytic Geometry, discovered by René Descartes (1637) and (2) the Infinitesimal Calculus, the contribution of Newton and Leibnitz to the learning and technical proficiency of man.

I have already mentioned on page 108 that the path of a cannon-ball, or, for that matter, the motion of any object is most readily studied by a graphical presentation of that motion. Nowadays graphs are very common. We see them even in the newspapers when things like the fluctuation of the price, say, of wheat is discussed. But it took nothing less than the invention of Analytic Geometry to put this simple and powerful device at the service of man.

If a body travels along a curved path, it does so under the action of a force exerted upon it. If the force suddenly stops, the moving object continues nevertheless to move, not along the curve, however, but along the tangent to that curve at the point where the object was when the action of the force ceased. Thus, in the study of motion, it is important to be able to determine the tangent to the path at any point of that curve. The resources that mathematics had to offer up to the middle of the seventeenth century were insufficient to solve that apparently simple problem. The differential calculus provided the answer.

The calculus provides the tools necessary to cope with the questions involving the velocity of moving bodies and their acceleration, or pick-up. The ancients had only very hazy notions about these concepts. The unaided imagination seems to find it very difficult to handle them successfully. The methods furnished by the calculus take all the sting and all the bitterness out of them. When velocity and acceleration are presented to students of mechanics who do not have the calculus at their disposal, these notions are still explained in terms of the calculus, in a roundabout fashion.

Our own age is confronted with technological problems of great difficulty. The mathematical tools they call for were not in existence at the time of Newton, two centuries ago. The airplane alone is sufficient to make one think what a variety of questions of an unprecedented kind had to be answered, what complicated problems had to be solved to enable the flier to accomplish all the wonders of which we are the surprised and admiring witnesses. The difficulties of constructing the airplane wings necessitated the concentration of mathematical talent and mathematical information that hardly has any parallel in history.

As has been pointed out, sea-faring called for the solution of many problems. However, a ship sailing the high seas has one important feature in common with a vehicle traveling on land: both move on a surface. From a geometric point of view the problems related to their motion are two-dimensional. An airplane that roams in the air above is engaged in three dimensional navigation. The geometrical aspect of flight belongs to the domain of Solid Geometry, and the problems connected with it are thus much more difficult, other things being equal.

Mathematics plays an enormous role in the field of social problems, through the use of statistics. I have already pointed out the value of mathematics in connection with the collecting of taxes, at earlier stages of civilization. The functions of a modern government are vastly more complex, more varied, and applied on an enormous scale. The variety and scope of problems modern government is interested in can be gleaned from the questions

the citizen is asked when he receives the census blank, every ten years. To study the wealth of information that is thus gathered on millions of blanks is the function and the task of the census bureau. The inferences that can be drawn from these data are as involved as they are far-reaching in their applications. Such a statistical study requires a wide range of mathematical equipment, from the most elementary arithmetic to the most abstruse branches of mathematical analysis. If one thinks of the new functions of social welfare that the government has taken on, like social security or old age pensions, as well as of those that are in the offing, like health insurance, and the millions of individuals that these services cover, one is readily led to the realization that the intelligent dealing with these services sets before the government new statistical problems of vast magnitude.

The government is not the only social agency to use statistics. Far from it. Insurance companies have been using statistics for a long time. Banks and other organizations which study the trends of business arrive at their conclusions and their predictions by statistical analysis. The study of the weather raises many very difficult statistical problems. Statistics is used to determine the efficiency of the methods of instruction in our public schools. This list could be made much longer and become boring by its monotony. As it is it will suffice to convey the idea of the all-pervading role this branch of applied mathematics plays in our modern life.

I have alluded several times to the fact that during the course of the centuries mathematics was called upon to provide solutions for problems that have arisen in various human pursuits, for which no solution was known at the time. This, however, is not always the way things occur. In many cases the reverse is true. When the need arises and the question is asked, mathematics reaches out into its vast store of knowledge accumulated through the centuries and produces the answer. The astronomer Kepler had before him a vast number of observations concerning the motion of the planets. These figures were meaningless until he noticed that they would hang nicely together if the planets followed a path of the form which the Alexandrian Greek Apollonius called an ellipse. Another plaything of the same Apollonius, the hyperbola, came in very handy to locate enemy guns during World War I, when the flash of the gun could be observed twice.

This readiness of mathematics goes much further. Various branches of science, when they pass and outgrow the purely descriptive stage and are ready to enter the following, the quantitative stage, discover that the mathematical problems which these new studies present have already been solved and are ready for use. Thus, Biology has in the last decades raised many questions, answers for which were available in the storeroom of mathematics. At present the scope of mathematics used in "Mathematical Biology" exceeds by far the mathematical education which our best engineering schools equip their graduates with. A similar tale can be told of psychology, economics, and other sciences.

I have tried to point out the close relation of the mathematics of any period of civilization to the social and economic needs of that period. Mathematics is a tool in the work-a-day life of mankind. It is closely connected with the well-being of the race and has played an important role in the slow and painful march of mankind from savagery to civilization. Mathematics is proud of the material help it has rendered the human race, for the satisfaction of these needs is the first and indispensable step that must be taken before higher and nobler pursuits can be cultivated. The Hebrew sages of yore said: "Without bread there is no learning." The Russian fable writer Ivan Krylov put it in a more crude but striking way: "Who cares to sing on a hungry stomach?"

But mathematics does not limit its ambition to serving utilitarian purposes. "Man does not live by bread alone." It would be utterly erroneous to think that the development of mathematics is due solely to the stimulus it received from the outside, or was always guided by the needs of the moment, or of the period. Rather the contrary is true. To be sure, the mathematician, like any other scientist, is not unmindful of those needs and is not indifferent to the acclaim that would be his if he succeeded in supplying the answer to a pressing question of the day. But, by and large, the development of mathematics is mostly stimulated by the innate human curiosity and by the pleasure derived from the exercise of the human faculties. The only extraneous element is perhaps the approval of a small group of likeminded people. Beyond that the reward that the mathematician receives for his efforts consists in the pains of creation and the joys of discovery. If his labors can be useful now, so much the better. If they are not, they have a chance to be so in the future, perhaps in the distant future. "Without learning there is no bread," to quote the other half of the saying of the Hebrew sages. But to the mathematician his labor is primarily a labor of love, and the product of his labor—a further addition to the stately edifice that he calls the Science of Mathematics.

In the cultural history of mankind mathematics occupies a unique place. It is the model science. Other sciences try to approach its objectivity and its rigor. Mathematics was the first to point out that any branch of learning must start out with a certain fund of data that must be accepted as given and true, and which cannot be further analysed. When stated in this form, this idea seems so obvious that it borders on triviality. But it represents the product of a century of keen analysis and much labor. It led to the postulational approach to mathematics. The mathematician stopped worrying what a point is or what a straight line is. He simply says: "these are things that I accept as my original stock in trade." Neither is he trying to prove that two points determine a line, or that two lines can be parallel, or that through a point a line can be drawn parallel to a given line. This is simplyso much more stock in trade. When he has accumulated enough of such, he is ready for action. He is ready to start building his science, in this case, of geometry. But how much is "enough"? Could there be too much of it? It is much easier to ask these rather obvious questions than to answer them. They were the object of much debate and are still one of the items of unfinished business in Mathematics.

Algebra

Much to my dismay and wrath
A truly different kind of Math
Was introduced to me this year,
To make me dread and weep with fear.

And as each night my brains I rack,
And say aloud, "My aching back!"
I wish and wish that I could be
Improved in my ability
To make my algebraic mind,
A better, more efficient kind.

MARY ANN WOOD
Grade 9
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Merion, Pa.

The Engineering Staff's Responsibilities and Opportunities in the Improvement of Learning and Teaching of Mathematics in the Secondary School[1]

By James H. Zant

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The fact that many prospective engineers are not properly prepared in mathematics has been recognized for a long time. Some mathematics teachers profess to see a steady deterioration in the quality of the high school student as he comes to the engineering college. At a conference of college and high school teachers of mathematics held in New York last year the following conclusions were drawn.

1. A vast majority of the high school people believe that their prime function is to educate all the youth and not merely to prepare a relatively small percentage, although large in numbers, for college. Therefore, in mathematics, technique is not as essential or desirable as it was, say, ten years ago.
2. The range of mathematical topics taught in the high schools today attempts to encompass the field from arithmetic through topology, including calculus.
3. The high school teacher says his job is to emphasize concepts rather than techniques.[2]

This brings out the first point I would like to make in this discussion, namely:

1. *It is the responsibility of members of engineering staffs to know the view points and problems of the secondary school, especially regarding science and mathematics.*

Much of the current misunderstanding and criticism is due to the fact that engineers seem to assume that the high schools exist to prepare a small percentage of students to enter the college of engineering. The fact that this same criticism could be made of almost any other group of college specialists does not release the engineers from this responsibility.

In meeting the demands of the people at large the secondary schools of the United States have a vast and complicated problem and no one group has a right to expect special treatment. Hence those of us interested in engineering education, and in mathematical education as one of its most important prerequisites, should also make it our business to know and consider the various problems of the public and private secondary schools so that we can make worthwhile suggestions regarding the improvement of the teaching of mathematics for prospective students.

2. *The engineering teacher should know the mathematical needs (both skills and concepts) of the engineering student.*

Few, if any, engineers fail to stress the need of mathematics by their students. However, when dealing with the specific topics which are needed by the student *before* he starts the study of engineering the teacher, or the engineer, is often vague. The secondary schools have a right to expect specific advice if they in turn are expected to change their curriculum and segregate their students into special groups.

The article by Professors Miller and Roth, referred to above, gives such specific advice to secondary schools. Some of their recommendations follow:

1. The courses in algebra should be stepped up so that a higher attainment of skills can be accomplished at the end of each. Some of the techniques in intermediate and advanced algebra should be placed in the elementary course and then reviewed and extended in later studies. One of the ways to accomplish this goal is by a study of functional form, another is by eliminating extraneous topics.
2. We believe that—the schools are not

[1] Read on June 18, 1947, before the Secondary Education Division of the American Society for Engineering Education at Minneapolis.

[2] Frederic H. Miller and Sidney G. Roth, "A Report on Mathematics Preparation for Engineering Colleges" *The Journal of Engineering Education,* Vol. 37, No. 8 (April, 1947) p. 628.

teaching concepts any better than techniques and consequently the whole course suffers. In the regular courses in mathematics from elementary algebra through trigonometry and advanced algebra, the many gaps of techniques and concepts should be closed. This can be accomplished by leaving for honors courses extended discussions of theories of infinities—topology, etc. The time saved by the omission of these topics can be utilized by practice in manipulation of higher level; by more careful treatment of derivations so that the students learn the nature of a proof; by a systematic study of deductive reasoning as opposed to inductive reasoning throughout the high school curriculum not only in geometry, etc.

4. Remove the section on spherical trigonometry from the trigonometry course. Its inclusion during the war period was based on the specious argument that spherical trigonometry was useful for navigation. Perhaps the validity of such a statement would be unquestioned if the war referred to was World War I. The modern methods of navigation are based on relatively extensive tables and devices for computation. In any event the dire need for navigators is past!

6. Re-introduce inverse trigonometric functions and emphasize radian measure. Use both more frequently.

8. Replace Horner's method with the method of successive approximations or at least teach the latter along with the traditional. By doing so other types of equations can be treated, e.g., exponential and trigonometric.[3]

We are not concerned at the moment with the general validity of these suggestions. They are specific. They tell the high school teacher of mathematics exactly what the writers think can be done to improve their teaching from the standpoint of the prospective engineering student. However, this advice is not given in a narrow academic manner. Professors Miller and Roth affirm their belief that the college should not dictate to the high schools what is to be taught for all students but only insist that the mathematics given to prospective engineers and scientists should conform to the proven needs of the students. It is just as important for the engineering teacher to be familiar with these needs and be able to advise and discuss these problems intelligently with high school teachers and pupils.

3. *It is also the responsibility of the engineering staff to be able to tell the prospective engineering student and his teachers what his prospects of success are if he enters the study of engineering.*

This problem of predicting a student's success, or more accurately setting up the qualifications which he must have in order for it to be possible for him to succeed, has been studied for several years in connection with the Measurement and Guidance Project in Engineering Education under the able direction of Dr. D. W. Vaughn. Progress is being made toward tests which students may take in high school or upon entering college which should enable the College of Engineering to predict his probable success.

The secondary schools and the students will naturally expect the teachers of engineering classes to explain and interpret the results of these tests. A knowledge of the content and results of these tests will also make it possible to give more intelligent advice on what mathematics should be taught in the high school to those students who expect to study science and engineering.

4. *The engineering staff is also in an excellent position to discuss mathematical needs and values with teachers and students in the high schools.*

Such discussion will be welcomed and beneficial to prospective engineering students. It will be an opportunity to present the mathematical needs and requirements and should help in pointing the way toward a better type of training in this field. It will be necessary, of course, to have a complete knowledge of the points discussed above in order for these contacts to be fully successful. The secondary school students and teachers will respect the engineer's discussion and opinions if he has such definite knowledge for use with them.

5. *The engineering educator should know something about the range of mathematics used in the various fields of engineering.*

A thorough knowledge of mathematics far beyond calculus is rapidly becoming necessary in order to keep pace with the

[3] Miller and Roth, *op. cit.*, pp. 635–637.

advances of modern science and engineering. The amount and type of mathematics needed varies widely with the various fields of engineering and few, if any, engineers can be expected to have a working knowledge of all of it. However, the teachers in engineering colleges should have some concept of what is needed and be in a position to advise with students regarding the mathematics they will ultimately have to learn if they are to be successful in modern engineering.

The Mathematics Division of the A.S.E.E. has spent most of its time during this meeting discussing the types of applied mathematics needed by some of the branches of engineering. This mathematics is different, often considerably so, from the classical mathematics taught in many departments of mathematics. For example, the treatment of differential equations should go beyond the usual rather detailed study of special types of equations that rarely, if ever, arise in engineering work and which "are given undue attention merely because they may be subject to 'elegant' methods of solution (which often depends on tricks or ingenious devices), or because they have geometric interpretations that are of interest (at least to some mathematicians)."[4] It is desirable to include "some experience with differential equations that stem from physical problems and which cannot be solved in simple closed form, so that approximation methods or infinite series solutions must be employed. In addition—since many physical problems with physical applications involve more than one independent variable, some work with linear partial differential equations should be included."[5]

There is a growing need for many other phases of mathematics in the practice and study of engineering and if engineering educators have at least a speaking knowledge of this mathematics, they will be able to contribute definitely to improving the attitude of the prospective engineering student toward this important and necessary subject.

6. *Proposals for the improvement of secondary school mathematics.*

Engineers will be interested in some of the agencies and committess which are studying the secondary situation in mathematics teaching and making suggestions designed to improve it. Three of these committees are: The Commission on Post-War Plans of the National Council of Teachers of Mathematics. The Cooperative Committee on Science and Mathematics of the A.A.A.S., and the Coordination Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics. These groups have made three basic types of suggestions regarding the improvement of teaching and learning in mathematics.

1) *The recruitment and preparation of teachers.*

Teacher shortages have received so much national attention that no general discussion need be given here. It is probable that shortages in Mathematics are as large or larger than in other fields and for the same reason the number of temporary certificates is also high. It goes without saying that many of these emergency certificate holders are teachers who have previously been unable or unwilling to make minimum preparation for teaching jobs.

Some are advocating a special curriculum in our colleges and universities for the training of mathematics teachers. While this may not be the answer to the problem it is clearly the duty and responsibility of the college departments to try to interest their students in teaching and to modify the curriculum, if need be, so they may be properly trained.

2) *Some reorganization of the public school curriculum to give the prospective student of science and engineering a better background* is proposed by the committees as well as individuals. The Commission on

[4] Fredric H. Miller, "Mathematics Beyond Calculus for Engineering Students" *The Journal of Engineering Education,* Vol. 37, No. 4 (Dec. 1946, p. 331).

[5] *Ibid.*, p. 332.

Post-War Plans makes two basic proposals. First, that it be the responsibility of the entire public school from Grade 1 through 12 to see that every student who is capable attains "functional competence in mathematics." The Commission made no attempt to give an explicit definition of "functional competence." However, progressive mathematics teachers have known for many years that pupils were leaving the schools at graduation time with definite shortages in their mathematical knowledge. The Commission believes that it is the responsibility of the whole school to do something about it.

Second, beyond "functional competence," usually attained by the study of mathematics through Grade VIII, the mathematical offering in the high school should be "double-track", algebra, geometry, etc., for some and general mathematics for the rest. The general mathematics course is recommended so that the larger group who are not interested or capable of pursuing a scientific course may have this opportunity to complete the requirements for functional competency and also give them the mathematics needed so that they can render semi-skilled service in industry, the larger business concerns, small private shops, and businesses and the like. The organization of these courses in general mathematics also has the function of eliminating the group of students who lack interest and ability from the courses in sequential mathematics (that is, those which have a traditional, logical order, algebra, geometry, trigonometry, advanced algebra, etc.).

It is recommended that the sequential courses should be reserved for those pupils who, having the requisite ability, desire or need such work. It is necessary of course that prospective engineers and scientists should take these courses. The main advantage of this reorganization is that the weaker sudents will be eliminated and the courses may then be organized with the needs of this select group in mind.

3) *Much is being said about guidance for the secondary school student and attempts have been made to determine the mathematics he will need to succeed in engineering.*

The Commission on Post-War Plans has published[6] a pamphlet on Guidance for students and teachers interested in secondary mathematics and beyond. While this pamphlet deals with many fields other than engineering, it discusses the mathematical needs for a student who expects to enter engineering and makes definite suggestions about the mathematics he should take in high school, and the like. It is written in language which is understandable to the high school student. The information has been checked thoroughly.

All of these reports and suggested reorganizations point toward an awareness of the problem of poor preparation on the part of the beginning engineering student. It does not mean that the problem will be solved in short order. Many of the difficulties and suggestions as well have been pointed out and discussed for more than a generation. Yet the organization of high school mathematics in the majority of our high schools has changed little since the turn of the century. It is true that not as large a percentage of the enrollment chooses courses in mathematics. However due to the vast increases in high school attendance, the intelligence level of those who do take the courses has been so reduced that the courses given are much weaker than formerly. A few schools will change their organization to conform to suggestions made by the various committees, but before we can expect any general reorganization all interested parties, mathematicians, scientists, secondary school men, and engineers alike must know the program and use all of the influence we have to get it accepted by the secondary school people.

[6] *Guidance Pamphlet in Mathematics.* Single Copies 25¢ each postpaid. Orders of 10 or more 10¢ each postpaid. Send all orders to THE MATHEMATICS TEACHER, 525 W. 120th St., New York 27, N. Y.

A Laboratory for Meaningful Arithmetic

By Foster E. Grossnickle
State Teachers College, Jersey City, N. J.

The arithmetic classroom of today does not fit into any one pattern. In some cases it is primarily a "recitation period." In this case the teacher checks on the accuracy of the work of the pupil. In some cases the classroom is a period for assigning work and for a discussion of problems or processes which are not understood. Finally, the classroom is a laboratory in which the pupils manipulate material to discover principles and to arrive at valid conclusions. In this case the teacher asks leading questions but the pupil finds the answers by using his materials and equipment. He does not memorize a series of unrelated facts and give answers which are meaningless to him. After he understands or discovers a principle, he may use several periods practicing the application of that principle. The classroom takes on the appearance of a workshop. Under such conditions the pupils may be working in groups which have a common problem or interest for them. The work is largely individualized as each pupil must discover for himself the underlying principle to be learned.

If the classroom assumes the nature of a laboratory, it is essential for the teacher to have apparatus which she and the pupils may use. Too frequently the only equipment in an arithmetic classroom is a blackboard, chalk, eraser, paper, and pencil. This kind of equipment is conducive for producing a pupil whose knowledge of number comes from pencil manipulation and not from a knowledge of ideas. The former results in the acquisition of skills but the latter leads to meaningful arithmetic. Number is not a tool subject but it is a systematic way of representing quantities and of showing quantitative relationships. There is little opportunity for discovery and development of insight into number when the only material a pupil has is paper and pencil. He should use these things after he has discovered the procedure or process and not learn the process in a mechanical manner by working a large number of examples.

A Teacher's Kit for Arithmetic

It is not possible to make a complete list of manipulative materials for the teacher of arithmetic. The number of items included should be augmented from time to time depending upon the grade level and the topics presented. Since a complete list of laboratory materials for arithmetic cannot be formulated, a minimum list should be provided. The minimum list may be designated a *teacher's kit* for arithmetic. The materials in this kit are effective and essential for making number meaningful to the pupil, especially for the pupil who is average or below average in discovering number relationships and in developing insight into number.

The arithmetic kit should contain two kinds of material. One kind consists of items which are easily purchased at dime stores or are readily secured from the home. The other kind contains items which are made specifically for enriching the teaching of concepts and principles in arithmetic.

The list of materials useful for the arithmetic classroom which the teacher may secure at various stores or from her home includes such items as a foot ruler graduated to eighths or sixteenths, a yardstick, tape measure, milk bottles of different sizes, measuring cup to show halves, thirds and fourths, measuring spoons, clock face with both Arabic and Roman numerals, and some of the most widely used weights and measures in both dry and liquid measures.

The materials specifically designed to make arithmetic meaningful include such

things as an abacus, place-value pockets, fact-finders, fraction discs, and fraction charts. The abacus and place-value pockets are used to show the meaning of the decimal system of number. Place-value pockets contain three compartments made of cloth or wood for showing ones, tens, and hundreds. A fact-finder is a small rod containing 20 moveable beads. These beads are used to represent any fact in addition and subtraction and those facts in multiplication and division in which the product or the dividend does not exceed twenty. Fraction discs are circular blocks about 10 inches in diameter which show a whole, halves, thirds, fourths, sixths, and eighths. The fraction chart shows the equivalence of fractions in the family of halves, fourths, eighths, and sixteenths, and in the family of thirds, sixths, and twelfths.

The few items described are typical of the kinds of materials which may be made specifically for presenting certain phases of number meaningfully to the pupil. The teacher should add to her laboratory equipment as she has occasion to introduce different topics. The materials mentioned are suitable for showing the basic processes with integers and for developing the fraction concept. Other materials are needed for decimals, per cent, and many other topics in the upper grades.

The ideal program should provide at least three different stages or steps for the presentation of a new process or principle. These three sequences are:

1. Teacher-pupil demonstration
2. Pupil discovery period for himself
3. Practice material for mastery of the process

The application of these three stages for a development lesson may be illustrated by showing how to present the reduction of fractions. The teacher designates a pupil to demonstrate with the fraction discs the equivalence of such fractions as $\frac{2}{4}$ and $\frac{1}{2}$ or $\frac{3}{6}$ and $\frac{1}{2}$. This stage is the teacher pupil demonstration. Now each pupil has manipulative material to correspond to the fractional discs so that he demonstrates the equivalence of each set of fractions presented in the demonstration. In case he does not have such material he cuts paper to represent the amounts, or he makes drawings to show the fractions. He continues this process until he is ready to discover, with the aid of leading questions by the teacher, the principle that both numerator and denominator of a fraction may be multiplied or divided by the same number without changing the value of the fraction. Finally, he needs practice to master the process so that he is able to reduce to lowest terms the most widely used fractions. According to the above sequence of events, practice is an essential part of learning, but this practice is not given until the pupil understands the material to be practiced.

Two Phases of a Meaningful Program in Arithmetic

The use of manipulative materials alone will not insure meaning and understanding in arithmetic. There are two phases of a meaningful program. First, there must be social applications and usages of number. This phase is known as the *significance* of number. And second, there is the *mathematical phase* of number which consists in seeing and discovering relationships between quantities.

The significance of number, or its social applications, may be illustrated by a class which made candy for a candy sale. In order to make the candy, it was necessary to follow a recipe that used fractions; hence, the social situation served as a medium for introducing fractions. The class had to multiply and divide fractions by whole numbers or whole numbers by fractions. The problems encountered represented a social situation for fractional usge.

The pupils used measuring cups and weights (ounces) for finding fractional parts of a whole. The manipulative materials enabled the pupils to find the answers to certain problems. Then the

pupils learned how to find the answers by computation or by learning the algorism. A program of this kind may or may not be meaningful. The meaningful part of the operation depends upon the insight which the pupil has of number relationships.

A recipe may call for $\frac{3}{4}$ cup of an ingredient. If three times the recipe is made, the pupil may learn how to multiply a fraction by a whole number in a mechanical manner. In this problem the result is $2\frac{1}{4}$ cups. The pupil may measure $\frac{3}{4}$ cup three times and find the total amount to be $2\frac{1}{4}$ cups. This operation assures him that his computation is accurate but even under such conditions the operation is not meaningful from a mathematical point of view.

The typical housewife who plans to use three times a recipe which calls for $\frac{3}{4}$ cup of an ingredient will measure $\frac{3}{4}$ cup three times instead of measuring $2\frac{1}{4}$ cups. She does this for two reasons. First, she is not sure she remembers how to multiply fractions (assuming that she recognizes this situation as an example in multiplication), and second, she has no mathematical meaning of the operation. In order to execute her culinary art successfully, she embarks on a procedure which does not require any mathematical computation or mathematical thinking. Mathematical understanding is a thinking operation.

It is possible for a pupil to multiply the example $3\times\frac{3}{4}$ correctly and to check the result with measurement and yet the operation may not convey mathematical meaning to him. Now we are confronted with the problem of what constitutes this meaning. Mathematical meaning results from seeing relationships among numbers and it enables the pupil to have assurance that the result is correct because of his thought process. A pupil may multiply $3\times\frac{3}{4}$ in a mechanical manner by memorizing the steps in the solution. He objectifies the answer by measuring $\frac{3}{4}$ cup three times. In all of these processes he does not use any mental operation which is based on number relationships. He verifies the result by concrete and specific illustrations. Number is abstract and it is general instead of specific. It is entirely intellectual and not in the realm of the senses.

The product of 3 and $\frac{3}{4}$ is mathematically meaningful to the pupil when he can verify the result by quantitative thinking. He may know that $3\times1=3$, but since $\frac{3}{4}$ is $\frac{1}{4}$ less than 1, the product of 3 and $\frac{3}{4}$ will be $\frac{3}{4}$ less than 3, or $2\frac{1}{4}$. He may reason that $3\times\frac{1}{2}$ is $1\frac{1}{2}$, and since $\frac{3}{4}$ is $\frac{1}{4}$ more than $\frac{1}{2}$, the product of $3\times\frac{3}{4}$ will be $\frac{3}{4}$ more than $1\frac{1}{2}$, or $2\frac{1}{4}$. He may discover that $4\times\frac{3}{4}$ is 3; hence $3\times\frac{3}{4}$ will be $\frac{3}{4}$ less than 3, or $2\frac{1}{4}$. Finally, he should know that he can verify the result by finding the sum of $\frac{3}{4}$, $\frac{3}{4}$, and $\frac{3}{4}$. In each of these cases the pupil does not depend upon objective material or mechanical computation for the result; however, he uses the manipulative material before making the insightful discoveries about the product. He understands number relationships to verify his answer. He knows the answer is valid not because the computation is correct but because he uses a pattern of mathematical thinking to verify the result. Operations of this kind require a knowledge of mathematical meanings and understanding of the number system.

Failure to give due consideration to either or to both significance and mathematical meaning of number has resulted in a faulty program in arithmetic for many years. The social significance of number gave rise to the *social utility theory* of teaching arithmetic.

The advocates of the social utility theory of arithmetic assume that a worthwhile project is the essential element for providing a meaningful program in arithmetic. The emphasis in this kind of a program is on the *felt need* of the learner and the adult usage of a given process. The criterion of felt need applies to the pupil's immediate need for the solution to a given problem. If he wishes to make a boat, then the necessary arithmetical skills used in completing the project are stressed.

The criterion of adult usage applies to shaping the content of the curriculum. The proponents of this criterion limit the teaching of arithmetic to those things which are used in the business world. This group believes in reducing the amount of content to be taught so as to make it 100% accurate by intensifying the drill program.

The social utility theory is not tenable because it gives no emphasis to meaning and understanding of number. The criterion of felt need emphasizes extrinsic interest of the pupil which is transitory instead of intrinsic interest which is based on insightful experience. Limiting the amount and kind of arithmetic to adult usage has been beneficial in eliminating much of the useless material from the curriculum. On the other hand, the method of approach of the believers in this criterion represents the epitome of the drill theory of teaching number. Thus, we may conclude that the social utility theory of teaching arithmetic lacks the leaven to make an effective program in the subject.

Little need be stated about the folly of a program in arithmetic which stresses the mathematical side of number and neglects its social usage. "Learn a thing in its setting" is a psychological truism. A mathematical meaningful part of a program in arithmetic is the intellectual or the thinking part of the program. Manipulating materials and computation may be mechanical but the interpretation of the results constitutes mathematical thinking. Thus, it is impossible to dissociate the significance of number from the mathematical phase of it in a satisfactory program in arithmetic. The one gives meaning to the other.*

The social phase of number provides the setting for the problem. Manipulative materials enable the pupil to objectify the result. The mathematical phase adds meaning to the social situation because the pupil now sees quantitative relationships in the problem or situation. It follows, then, that significance and mathematical meaning supplement each other providing the pupil uses manipulative or other visual aids which enable him to discover the interrelations between these two traits. A program in arithmetic which gives due consideration to both significance and mathematical meaning exemplifies the meaning theory in the teaching of arithmetic. A laboratory of manipulative materials is essential for executing a program of this kind.

* This point of view is different from that presented by Wheat, "Fallacy of Social Arithmetic," THE MATHEMATICS TEACHER, 39: 27–34; January, 1946.

Guidance Pamphlet in Mathematics

Here are two more interesting comments on *A Guidance Pamphlet in Mathematics*

I appreciate the copy of THE MATHEMATICS TEACHER containing the report of the special commission. I am mentioning it in *Occupations* which is read by the vocational counselors of the country.

Harry D. Kitson
Professor of Education
Teachers College, Columbia University

Very many thanks for the good looking *Guidance Pamphlet in Mathematics* for high school students. I think you have set a standard which other content fields will try to meet. I am so glad that it is available in separate pamphlet form. It will be exceedingly useful I know.

Ruth Strang
Professor of Education
Teachers College, Columbia University

The above pamphlet may be secured at a cost of 25¢ each postpaid, or at 10¢ each in lots of 10 or more, from THE MATHEMATICS TEACHER, 525 W. 120th Street, New York 27, N. Y. It will be of great assistance to the office of THE TEACHER if these orders are accompanied by the correct amounts involved.—*Editor*

A Super-Plan!! Dedicated to All Security Worshippers*

By C. W. Watkeys, *Rochester, N. Y.*

A new party is being formed which will be called the Super Ultra Progressive Party, hereafter known as the SUP Party, pronounced the Soup Party as in "Soup of the Evening, beautiful, beautiful soup."

The fundamental goal of the SUP Party is the establishment of Automatic Economy to take the place of Planned Economy. Under Planned Economy the battle of words has reached colossal heights. With as much repetition as a Tibetan Prayer-wheel the respective camps blare forth stridently the terms "avaricous facists," "reactionary capitalists," "autocratic communists," "self-centered, short-sighted trade unionist," words, words with little meaning. To eliminate this strife we must pass through the puerile stage of Planned Economy to the final stage of Automatic Economy.

To attain automatic economy, our thinking must be hyper-progressive and not falter at apparently insuperable obstacles. Automatic Economy will automatically result if we can gain control of the seasons and of the weather! Mark Twain pointed the way when he remarked that everybody talks about the weather but nobody does anything. The SUP-ers will be the first to do something about the weather, impossible as it may seem, and the party emblem will be a whirling cyclone inclined at an angle to the perpendicular.

Careful analysis of our economic confusion leads to its fundamental cause, namely, the axis of the earth is inclined to the plane of the earth's orbit around the sun. To put the matter simply, the rays from the sun fall on a hemisphere of the earth and as the earth rotates around the sun this lighted hemispherical surface swings up and down causing the seasonal variation and the fluctuations of the weather.

If the axis were straightened so as to be perpendicular to the plane of the orbit the hemisphere of light and heat from the sun would extend from pole to pole throughout the year. Day and night would each be twelve hours long the year round. Hence would follow our first great triumph. We would eliminate daylight saving!

The average daily temperature would decrease from the equator to the poles but the range of the variation of temperature in any one place would be small. Hence each person could choose a locality with the preferred average temperature which would vary but little throughout the year. This would simplify the clothing problem!

Since the atmosphere above any zone included between two sufficiently close parallels of latitude would pass through approximately the same cycle of change, rain and sunshine would follow each other in regular sequence. Thus crops harvesting being done by automatic machinery and distribution becoming routine, Automatic Economy would necessarily follow. The struggle from free economy through the simple idealism of planned economy to the

* This amusing sketch was published on October 19, 1946 by the *Times-Union* of Rochester, N. Y. It attracted considerable attention and was made the subject of favorable editorial comment. Throughout his distinguished career at the University of Rochester, Professor Watkeys has been interested in the improvement of mathematical instruction in the elementary and secondary schools. He has repeatedly acted as a mathematical consultant for the New York State Education Department. For many years Professor Watkeys has been the President of the Mathematics Club of the Rochester teachers of mathematics. He was recently elected a member of the Board of Governors of the Mathematical Association of America.

The ingenious master plan presented in this article, correcting at one stroke all the troubles of the globe, including those of education, surely deserves the close attention of the United Nations advisory committees, and especially of UNESCO.—Wm. Betz.

relief of Automatic economy will have been worth while.

The political effects would be tremendous. The life in any zone around the earth would be subject to the same conditions and in the course of time, international fears and jealousy would be eliminated and there would be no need for name-calling. In time each person in the world would automatically govern himself.

The effect on the fundamental emotions of each individual would be equally far reaching. With nothing to be gained by quarreling with a neighbor, pugnacity would disappear and its inseparable companion, fear, would follow.

The education of the individual would be much simplified. Each child would be taught how to push buttons of different colors. To be sure, it might be necessary to introduce refresher courses to drill on the difference between red and green buttons. But the mastery of this difference has always been difficult for human beings and is to be expected. In addition to this fundamental course of button pushing, the curriculum would contain four courses known as:

Games and Amusements 1
Games and Amusements 2
Opinions and Beliefs 1
Opinions and Beliefs 2

One of these courses would be given each year so as not to strain the child's mind. For those who still retained some curiosity, a reading course entitled: "Mistakes of the Past" will be provided.

After sufficient machines have been constructed for the Automatic Economy, there will be no need for courses in mathematics and science, which have been the principal causes of the inferiority complexes of the present. Children will need to know only so much arithmetic as is sufficient to check the change at a grocery store. In order to make it easier to acquire this material, it will be included in the Games and Amusements courses.

A hope long held by principals of schools will at last be realized. Report cards will contain only three measures of success, namely, Extraordinary, Colossal, and Unbelievable, though whether the last term is to represent a performance of the highest or lowest merit is still to be decided.

Thus at one great stroke, the inferiority complex and the intelligence quotient will be eliminated.

The new program will result in a great deal of leisure as it is expected that button-pushing will reduce the work week to a six-hour or less week. Hammocks for everybody will be spun by highly selected new species of silkworms and spiders, trained to spin according to hypermodern designs. As yet they are not union conscious.

Strikes will be eliminated as there will be nothing to strike for.

Capital will not be required as there will be nothing to do with it.

Politicians will become as dead as dodos as there will be no decisions required.

Along with the disappearance of positive and negative self-feeling, pugnacity and flight, curiosity will gradually dwindle away and all that will be left of the fundamental emotions will be attraction and repulsion sufficient to carry on the survival of the race.

To accomplish these great ends, the SUP-ers propose to call in for conference and planning, groups of scientists. We have some superlative publicity agents to arouse the interest and cooperation of the masses and to bring to their attention the work of the scientists.

Experts on astronomy, anthropology, biology, physiology, geology, paleontology chemistry, psychology, psychiatry, physics, cyclotrons, gyroscopes, radar, atomic bombs, jet planes, celestial dynamics, aerodynamics, statistics, and probability will soon be cooperating on this gigantic project with as much secrecy as possible!

Without divulging any details, it may be said that, in the rough, the plan is to sink a great cylindrical column of steel at

each pole, projecting above the surface sufficiently so that cyclotronic collars can be attached with rotations opposite to that of the earth, generating gyroscopic forces neutralizing those produced by the earth's rotation.

An armada of jet planes will be attached to the east side of the north pole extension and another to the west side of the south pole extension.

A new type of atomic bomb known as the repeating atomic bomb will explode on the west side of the north pole simultaneously with another on the east side of the south pole. They will continue to explode at regular intervals giving successive kicks to the projecting cylinders to help the pulls of the jet planes. When the axis is perpendicular to the plane of the orbit, the cyclotrons will be shut off, the bombs will cease to explode and the planes cease to pull. The gyroscopic forces due to the earth's rotation then will hold the earth in the resulting perpendicular position.

It is thus that the SUP-ers plan to extablish Automatic Economy.

If life under Automatic Economy proves to be too dull for the majority of people and pressure from the Hyper-ultra Conservative Party should develop, a committee to DEPERPENDICULARIZE the axis will be appointed who will experiment with the angle of inclination so as to determine that one which will bring the greatest good to the greatest number of people.

Anyone wishing to become a charter member of the SUP Party, please send his or her name to the author.

P. S. Note that the SUP Party will automatically remain in power since political government will be replaced by automatic government.

P. P. S. Let any old-fashioned conservative who believes in free economy and that mankind can advance only through meeting the challenge of changing social forces by means of individual effort and responsibility just think of those hammocks and all they imply for the advancement of civilization.

The following editorial appeared on the same page of the issue of the *Times-Union* that contained Professor Watkeys' article.—EDITOR.

THAT SUPER-PLAN

Professor C. W. Watkeys, professor emeritus of mathematics at the University of Rochester, no doubt had even more fun concocting his Super-Plan than we have in printing it. Our readers will enjoy it hugely.

At the risk of gilding the lily we might say that Professor Watkeys has something there. So many planners of riskless living ignore the basic fact that the eventfulness of human life and its unevenness are made so by the very conditions which make life possible on planet Earth.

Professor Watkeys, having been more closely linked with the physical sciences, spins a little fable that points the moral from the physical science angle.

One who leans to the biological sciences could easily go on from there focusing on the rhythm which marks so many stages of life and may even have been the agency of creation. No rhythm, no ups and downs, no risk! Maybe no life!

"Factoring Method" vs. Division

By William R. Ransom
Tufts College, Medford, Mass.

It is well known (by teachers and the better pupils) that if a product is zero, at least one of the factors is zero.

But long experience shows that most students cannot be taught to use this theorem properly. It may well be listed in the category of dangerous knowledge. The reasoning on which it depends is not heard often enough to graft it upon the student's mind, and the applications of it allow a jump from equation to result without a corresponding fundamental operation such as required in the other steps of a solution.

A better approach is to require that all results be obtained by operating (add, subtract, multiply, divide, take root) upon the given equation, rather than by putting down something under the justification of a theorem little understood. Also it is very important to emphasize that when it comes to division there is an exceptional number: we can add, multiply, or subtract any two numbers, and we may say further we can always divide *except that we cannot divide by zero.*

If the terms of an equation have a common divisor, the equation can be simplified by dividing it out. But mark the case where the common factor is an unknown:

$$(2x-1)(x+2)=3(x+2).$$

The proper procedure here is to say: The equation could be simplified by dividing by $(x+2)$, but it may be that such a division is impossible. Therefore there are *two cases*. Either,

$x+2=$ zero	or $x+2\neq 0$, and we
and we cannot divide by it	can divide by it, and obtain
But in this case $x=-2$, and this value checks in the original equation.	$2x-1=3$ whence $x=2$, which checks in the original equation.

If the "factoring method" could be dropped from elementary algebra courses, and the above division, two-case method could be substituted, our pupils would avoid many errors in trying to write down, off hand, solutions for such equations as

$$x(x+1)=6$$

and would gain an understanding of a fact about division which is much more important than the theorem cited in the paragraph at the head of this article.

NOTICE—The table of "Clerical Workers Employed" which appeared in "The Guidance Report of the Commission on Post-War Plans" on page 320 of the November 1947 issue of The Mathematics Teacher was in error. The correct table, which appeared in the reprint *A Guidance Pamphlet in Mathematics* read as follows:

Clerical Workers Employed
(1940 Census)

	Men	*Women*
Agents, miscellaneous	80,040	8,601
Attendants, physicians' and dentists' offices	1,387	27,922
Attendants and assistants, library	1,955	7,028
Ticket, station and express agents	37,363	2,154
Collectors, bill and account	38,374	3,316
Office machine operators	8,284	51,454
Shipping and receiving clerks	200,669	8,668
Stenographers, typists, and secretaries	68,805	988,081
Clerical and kindred workers, miscellaneous	1,134,933	630,471

AIDS TO TEACHING*

By

HENRY W. SYER
School of Education, Boston University
Boston, Massachusetts

DONOVAN A. JOHNSON
College of Education, University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B.3—*Making Par With Your Car*

The Travelers Insurance Company, Hartford, Connecticut

Free booklet on driving.

Description: This 20-page booklet is in the form of a quiz about driving. Although it is put out by the Travelers Insurance Company it has no advertising except their name mentioned very inconspicuously about four times. There are 35 questions concerning facts about accidents, traffic rules and regulations, physical laws, expert driving and general questions. Of these, 15 have some statistical or mathematical content.

Appraisal: If schools have definite courses in safety or automobile driving this booklet would be very helpful there; if not it is certainly up to all the other classes, physics and mathematics included, to do what they can.

B.4—*Numbers* (Booklet) by Maud Haddleton

Industrial Arts Cooperative Service, 519 West 121st Street, New York, New York.

Booklet, mimeographed, 33 pages.

Price, 50¢

Description: A heterogeneous collection of facts and drawings about figures, computing devices, money and weights and measures. It is carelessly put together with very little style or reason. There are sets of directions for making Napier's rods, a Babylonian tablet, and talley sticks. There are rather crude drawings of an abacus, ancient numerations, talley sticks, Napier's rods, Babylonian stylus and tablets, shell money, Egyptian weights, and an amazing end-piece of clowns and numbers. The one-page bibliography is inadequate.

Appraisal: The subject and purpose of this booklet is admirable, but the organization and poor presentation cause it to be worth less than the price asked. Even an outline which failed to give all the information, but covered the topics thoroughly with helpful references might be better. The idea of having plans for construction of models of the older devices is excellent but the directions are so poor that one would never be able to make the models unless he knew to begin with what was expected. The whole booklet aims at the elementary level, which is an excellent idea, but it fails to reach any well-defined aim at all.

CHARTS

C.2—*The River Mathematics*

Henry Holt and Company, New York, New York.

Chart showing relationships between branches of mathematics. Price, 10¢.

Description: Mathematics is pictured as a river with its source in primitive arithmetic ("primitive men count on their fingers"), dividing into two streams "geometry" and "number reckoning" which are separated by a mysterious island "cal-

* The first article in this new feature of THE MATHEMATICS TEACHER appeared in the February (1947) issue. We shall be glad to know whether our readers consider this a valuable help to them. As stated in the first article, the authors will be glad to receive any suggestions from our readers as to further teaching aids not already included that they consider worthwhile. —Editor.

culus, analytic geometry and trigonometry." The left side of the page is a column which lists fifteen mathematicians and twenty-two dates in chronological order. Tributaries to the main river are marked with the names of mathematicians also. This chart is reprinted from the book "The River Mathematics," written by A. Hooper and published by Henry Holt and Company.

Appraisal: Just as it stands this chart is a worthwhile addition to any mathematics classroom or laboratory. There is no reason, though, why the idea should not be used, expanded and improved. Similar, larger charts could be constructed by committees of pupils containing more information. If they are in color they could be framed as permanent additions to mathematics classrooms. By addition of caricatures or cartoons of the characters and incidents in the history of mathematics at the proper place the chart could be turned into a mural which might be painted permanently on a wall of the school.

EQUIPMENT

E.2—*Air-Age Research Maps and Globes*

Air-Age Education Research, 80 East 42nd Street, New York, New York

Maps and Globes (See below for descriptions and prices).

Description: This concern, sponsored by American Airlines, lists many maps, wall charts, motion pictures, film-strips, model plane kits, books, booklets, and lithograph prints concerning air travel; some of them would be useful in mathematics classes.

(1) *World Air Routes Wall Map;* Azimuthal equidistant projection based on geographical center of the United States, information includes chronological history of the progress of aviation, essay on map facts, airline time-distance chart, and total revenue miles flown by U.S. flag lines; 42″×50″, five colors; paper sheet, $1.00; on cloth with wooden rods, $5.75; on cloth with spring roller, $7.75.

(2) *World Age Desk Charts;* black and white chart based on World Age Routes Wall Map; 11″×11″; 25 maps for 40¢.

(3) *United States Air Transport Map;* Shows principal air routes, illustrated with sketches of people, products and points of interest; 31″×23″, Price, 10¢.

(4) *Air-Age Wall Maps;* Series of maps of the world centered on nine principal regions of the world showing how the world looks to a person in that area; 32″×40″, in two colors (except U.S. and South America—42″×50″, in four colors); following areas are covered: U.S., South America, Europe, Alaska, U.S.S.R., China, Australia, South Africa, and India; Paper sheets, $1.00; set of nine for $7.00.

(5) *Air Globe;* Names of places and their locations only are shown (latitude, longitude and boundaries are eliminated); rests in a cradle so that it can be turned to any position; measuring tape allows measurement of flying time and mileage over great circle routes; 12″ in diameter. Price $9.00.

(6) *Air-Age Plotting Chart;* blank charts for making air maps with any selected place as the center of an azimuthal equidistant projection; 46″×48″. Price, 75¢ each; 11″×18″, package of 50. 95¢.

Appraisal: These maps and globes are very beautifully manufactured. They are absolutely correct. The technical question of whether an azimuthal equidistant projection gives a true picture of the entire world as it "looks to a person in that area" must be settled before these maps can find widespread usage in mathematics or social studies classes. The cost may be slightly out of proportion to the relative importance of the mathematical topics covered by the use of these maps and globes.

One of the most neglected and useful applications of geometry is through geography. The mathematical treatment of maps, mapping, surveying, charts, and map projections is too geographical for the mathematics classes and too mathematical for the geography classes; therefore it falls between them. Since the technical mathematics is less well known than geography, it seems as though the mathematics

teacher might make the first move to introduce the subjects.

FILMS

F.4—*What Is Four?*

Young America Films, 18 East 41 St. New York 17, N. Y.
Teacher's Guide Available.
15 minutes, 16mm, sound.
Black and white or color.

Content: This film shows the mathematical meaning and use of the number four through a series of concrete, semi-concrete, and abstract situations. The meaning of four is illustrated by activities such as the number of milk bottles delivered by the milkman, the number of glasses of milk served by Tommy to his classmates, and the number of blocks used by Tommy and Ruth in constructing block towers. The combinations that make four and the relationship of four to one, two, three, five, and six are developed through the rearranging of circles, lines, squares, boxes into various number combinations and writing each combination in terms of numbers and words. The last half of the film illustrates and discusses some simple addition and subtraction facts and relates them to the symbols $+$, $-$ and $=$.

Appraisal: This film was produced for use as a teaching film in the primary grades. It would seem to the reviewer that the teaching of the meaning of four can best be done by the use of actual objects, activities, or applications in the classroom rather than by pictures of these objects. Although it may have some motivation value, too many concepts are discussed in 15 minutes for a primary class. However, the film has a definite place in the training of elementary teachers by showing a functional approach to the problem of developing number concepts. It should be useful in showing teachers how to provide a meaningful basis for addition and subtraction facts and in suggesting activities leading to an understanding of other numbers up to ten.

Technical qualities: Photography: good, effective use of animation.
Sound: satisfactory

F.5—*The Slide Rule I* "C" and "D" Scales
U.S. Dept. of Education
Washington, D.C.
2 reels, 16mm., sound, black and white.
Teacher's manual and film strip available.

Content: The purpose of this film is to teach the reading of the "C" and "D" scales and their use in simple multiplication and division problems. To do this a simplified rule is used which has all scales removed except the C and D scales. In order to simplify further the reading of the rule, secondary divisions are removed and the portion of the scale involved in a given problem is greatly enlarged. Animation is used to direct attention and to diagram the solution of problems. The film begins by giving the use of the slide rule and naming the parts. Only major divisions are used to solve simple multiplication problems like 3 times 4. After explaining and illustrating how to read the rule correct to three significant digits, multiplication and division problems such as 67.4 times 3.48 or 5.95 divided by 20.4 are solved. Considerable time is spent on learning to place the decimal point in the answer. In addition, a few simple problems containing both multiplication and division processes are solved.

F.6—*The Slide Rule II*, Proportion, percentage, squares and square roots
U.S. Dept. of Education
Washington, D.C.
2 reels, 16mm., sound, black and white, 1944

Content: This film shows how to use the slide rule for computing proportions, percentage, squares and square roots. The same methods of simplification and emphasis used in the first film on the slide rule are also used effectively in this film. The film begins by reviewing multiplication and division. This is followed by problems on proportions beginning with simple examples and proceeding to appli-

cations. The applications involve the computing of the height of a radio-antenna tower and the conversion of inches to metric units. Problems in percentage are solved by writing the problem as a proportion with one of the terms being 100. After explaining how to read the A and B scales, problems on squaring and square root are solved including an application to the solution of a right triangle. Explanations are given on how to locate the decimal point and read the answer. A review is given at the end of the film.

Appraisal: Although these films do not give the logarithmic basis of the slide rule, they are outstanding teaching films. The photography and commentary are excellent. By blotting out scales and numbers and superimposing scale readings the correct reading of the scale is assured. This scale reading is a difficult process to teach in slide rule instruction even with a large demonstration rule. The direction of attention to the proper part of the rule is well done by animation. This film follows the recommendations of educators in respect to the development of concepts and skills. For example, each process is followed by a careful summary of the steps involved. The selection of problems is appropriate, beginning with very simple problems and proceeding to more complex problems and applications. In addition, each process is diagrammed so that the student can visualize the method. It should be an effective film to supplement instruction in the use of the slide rule at any level of high school or college mathematics. It should be supplemented by more work on reading the scales and solving problems with slide rules. It could also be used for preview or review.

Technical qualities: Photography: excellent. Sound: excellent.
Commentary: appropriate and interesting.

FILM-STRIPS

FS.2—*Optical Illusions*

Visual Sciences, Suffern, N. Y.

Film-strip, 39 frames, $2.00, 1939.

Content: This film strip shows a large number of optical illustions illustrated by geometric figures. It shows how shape, size, and relationship between lines, circles and polygons may appear distorted by relating them to other lines, circles, polygons or shading. It mentions briefly examples of illusions in real life: such as the apparent motion of the moon through clouds. It also mentions applications of illusions such as dress materials designed to make the wearer look slim or tall. In order to assist the eye focus, small drawings with appropriate shading make it easy to see the illusions.

Appraisal: Optical illusions are an interesting sidelight in geometry and this film-strip furnishes the teacher with a fairly complete set. Although the drawings are complete, more interest could have been added by color photography, by more unique drawings, and more adequate descriptions. It could well have contained additional pictures of applications. The reviewer has found that drawings of optical illusions by students are excellent projects and can be shown either by display on the bulletin board or by opaque projection.

Technical Ratings: Photography: fair, inadequate instructional material
Level: Junior high school mathematics and geometry

INSTRUMENTS

I.2—*The Perspectograph* (Template for three-dimensional drawing)

Margaret Joseph, 1504 Prospect Avenue, Milwaukee, Wisconsin.

Single cardboard template for drawing diagrams. Price, 30¢.

Description: The cardboard template contains twelve elements of solid figures which can be traced and completed to make many figures. They include a triangular pyramid, an inverted triangular pyramid, a square pyramid, a hexagonal pyramid, a cube, a pentagonal pyramid, a circle (used for spheres), polar triangles, a cone, a cylinder, congruent 45° and 30–60°

triangles, and a spherical triangle. By including cut-out areas parallel to the bases of figures it is simple to draw frustrums and similar solids. It is stated that celluloid or transparent plastic templates will replace the present cardboard model by the second semester of this year and that larger ones for blackboard use will be available as soon as production costs diminish.

Appraisal: Since one of the objectives of solid geometry is usually the ability to draw three-dimensional figures free-hand, it would be unwise to use the perspectograph exclusively and never require free-hand diagrams. However even the cleverest draftsman sometimes needs help in maintaining correct proportions so this template should prove very helpful when the interest is on the principle of geometry or the proof and not on the drawing. It is unfortunate that some of the figures, such as the triangular pyramid, are drawn so that it is impossible to show auxiliary lines, for example, the altitude.

The cardboard is heavy and quite durable. Transparent celluloid would be a great advantage since it would allow the position of the drawing on the page and in relationship to other drawings to be seen before the tracing is made. It is doubtful how helpful large blackboard stencils would be.

I.3—*Laron Rule* (Blackboard Drawing Instrument)

Larson-Onley Products Company, Box 34, Lenox, Mass.
Blackboard ruler with gravity dial protractor and compass arm. Price $18.50.

Description: This instrument is really a ruler, protractor and compass in the same device. The dial mounted on the face of the wide ruler is activated by gravity so that angles from the horizontal can be read directly. This makes it possible to draw a line at any desired angle in a diagram, and therefore to draw parallels and perpendiculars at any desired angle to the horizontal. Circles may be drawn by placing the chalk in one of the holes in a pivoted arm attached to the rule. There is a clip in which problems may be placed for reference while working.

Appraisal: The equipment is very durably made. It has a handle which makes manipulation very easy. The gravity-protractor device works well and speeds up the drawing immensely. However, it is so small that it can be seen only by the one using the rule. The compass arm is handy, but awkward to use for the rule must be turned over to draw the lower half of each circle; the limitation of circles to five predetermined radii might prove cumbersome.

Actually the only new improvement which this rule offers is the gravity protractor. While this is very clever and useful, $18.50 seems rather high for such a novelty. Possibly there would be enough justification to buy one of these for each building to demonstrate the gravity principle. It might have its proper place in a mathematics laboratory or museum.

PLANS FOR CONSTRUCTIONS

PC.2—*A Two-Foot Globe*

Industrial Arts Cooperative Service, 519 West 121st Street, New York, New York
Booklet of directions; mimeographed; 14 pages. Price, 35¢.

Description: This helpful booklet lists the materials needed, the minimum tool list, preparations for making the globe, steps in making a working drawing, in making a template, in making the skeleton, in covering the skeleton, in padding out the sphere, in making the surface of the sphere, in marking the parallels and meridians, in drawing the oceans and continents, and in making the cradle to hold the sphere.

Appraisal: This booklet is very complete with every step carefully and clearly described; the drawings are very helpful and neatly done. Whether the finished globe is covered with a map of the world and then used by mathematics and social science classes together in their studies of

distances and great circles, or whether it is covered with important figures from spherical geometry and navigation for the use of the mathematics department alone, the building of it and the planning and measurement will be an exercise in practical mathematics well worth the time. By realizing the scope of knowledge needed to build this globe and the many levels of usage it can have, one can see that the project is suitable for any group from grades four to twelve.

SUPPLIES FOR LABORATORY WORK

SL.1—*Fotoflat* (material for mounting pictures)

Seal, Inc., Shelton, Connecticut

Comes in flat sheets in sizes from 1 15/16″ ×2 13/16″ (36 sheets for 15¢) to 16″×20″ (12 sheets for 80¢). Sixteen different sizes in all.

Description: This paper-like material, similar to the dry-mounting tissue sold by Eastman Kodak for mounting pictures, using heat and pressure rather than moist glue or paste, is placed between the picture and the cardboard upon which it is to be mounted. Heat from an electric iron or a special mounting iron causes the picture to stick to the mount.

Appraisal: This method of mounting is quicker, cleaner and easier than using paste or glue. It provides a flat, non-warping mount which causes all portions of the picture to adhere. On the other hand, it is more expensive and takes more time to mount a single picture since the iron must be heated before the mounting can begin.

SL.2—*Fotowelder*. (Special iron for mounting pictures)

Seal, Inc., Shelton, Connecticut.

Comes in four sizes, priced from $1.50 to $9.50.

Description: These irons are especially designed to make picture mounting easier when using the dry mounting tissue or fotoflat mounting process. They have handles which might be slightly easier to use than ordinary electric irons.

Appraisal: For schools which have large numbers of pictures to mount at a time and must do so quite frequently, the additional aid of special mounting irons may be necessary. Other schools can do the work well enough with ordinary flat-irons.

◆ THE ART OF TEACHING ◆

An Approach to Forming the Content of Non-College Preparatory Courses in Mathematics

By MARGARET MCALPINE MILLER
Grant Junior High School, Denver, Colo.

IF WE are going to have a curriculum in non-college preparatory mathematics in our high schools, certainly one of the first problems we will encounter is the problem of the course of study or the content of the courses.

This problem of content for non-college preparatory mathematics courses has baffled text-book writers in mathematics for many years. It is a difficult type of text-book to compose because no two communities, and hardly two schools or two groups of class personnel have exactly the same mathematical needs. The problem, then, of the course of study or content of the non-college preparatory mathematics courses lies almost wholly with the individual teacher. The teacher needs to organize her own content for her own situation.

How can this be done? How can a classroom teacher select the topics to be presented, collect the material and present it to the students in a form that they can handle? Where can this material be found?

The few suggestions given below are by no means the only solutions to the problem. They are listed as possibilities or guide lines along which a non-college preparatory mathematics course might be built.

Assure yourself, first, of the necessity of non-college preparatory mathematics courses in your school by examining the percentage of graduates from your school going to college. In most cases that figure will be less than twenty per cent.

The following are suggestions for aids in the selection of topics and a starter list of sources from which material may be extracted for non-college preparatory mathematics courses.

1. Examine the list of industries or businesses by which your seniors have been employed in the past. This data is not available in all schools but some are fortunate enough to have gathered this information from their seniors in the form of questionnaires. If your school has this information, analyze the positions, first, to determine the mathematical skills necessary for an efficient employee, and second, to determine any mathematical information of value in aiding job advancement. This data can be used in the classroom as a guide in the selection of topics as well as material to stress within your courses. For example, if a fair percentage of your graduates become mechanics or machine shop workers, it perhaps would be advantageous to include units on measuring instruments, i.e., the reading of the micrometer, etc. With this the teacher would have the opportunity to stress decimals and emphasize accuracy.

2. List the main industries in your town in which the graduates from your high school might be employed. Study these and perhaps with some aid from members of the organizations you can determine the mathematics necessary for work in them. Incorporate these requirements in your courses. This is a good source for problems with "insetting" appeal. The number of gallons of oil pumped out of an oil well per day would have much more meaning to a student in central Oklahoma than it would to one in rural Mississippi.

3. Examine the out of school jobs which your pupils hold. This, again, is a good source of "insetting" problems. It gives the teacher opportunity to bring a practical problem into the mathematics classroom that is a present need of at least some of the students.

4. A fertile source of material for content in non-college preparatory mathematics courses lies within the student's school activity. There are quantities of mathematical terms, mathematical operations and material for mathematics problems found in texts, references and teacher outlines of courses other than mathematics offered in your school. It does not take too long to run through a general science or home economics text and note the frequent use of mathematics. The job can be handled with greater ease if a check list is used and totaled at the finish of the investigation. In this manner the mathematical operations and terms that occur most frequently in courses other than mathematics can be found. This list will assist in the selection of topics and will give excellent source material. It will help the student tie together his material and is concerned with his present needs in mathematics. If time runs low in the process of looking up mathematical terms from the student's texts, the fast students might enjoy seeing that there is so much mathematics in their history books!

5. The most common source for extracting material is from a number of non-college preparatory type mathematics texts, selecting the material suited to your situation. A combination of this method and one of the foregoing ones should result in a satisfactory body of material usable in non-college preparatory mathematics courses.

How can all this mass of work be handled so that each student may have some form of guide?

1. Instructions can be written on the blackboard, but this becomes tiresome and the pupil has no copy after it is erased.
2. One or two copies of the instructions can be pinned on the bulletin board but this sometimes causes confusion if you have a very large class. Bulletin boards are generally well used for mathematical applications and illustrations from papers and magazines or display of class work.
3. Mimeographed sheets in looseleaf notebooks is a good method. The commercial department and the clerical staff are needed here.
4. Multigraph sheets pasted in a non-looseleaf notebook has the advantage over method number three in that the leaves do not fall out. Some believe that drawing and writing can be done with greater ease on the multigraph, saving some work for the office staff.

General Suggestions

Tables borrowed from the physics laboratory or the lunch room make map work and compass, protractor and instrument exercises easier, for everyone can work at the same time.

Ask science teachers, shop teachers, home economics teachers and even your friends in the history department for instruments to use as illustrations and for some practical experience exercises. Few schools are rich enough to buy two sets of instruments, so plan a borrowing scheme.

Keep a small notebook with you in which to jot down brilliant ideas as they occur. Applications sometime "pop up" in the queerest places! Keep the students on the alert to pick up applications and ideas for you.

Again it can be stated that these are not the only means to solve the problem of forming contents for non-college preparatory mathematics courses. They are meant only as suggestions as how it might be attempted in some situations. It takes time, work, and patience. It is not an easy approach, but non-college preparatory mathematics is not an easy subject to teach—it has been ignored and mistreated too long!

Please indicate whether your subscription is new or a renewal and, if possible, indicate the month of expiration.—Editor

The Duke University Mathematics Institute* 1947

By HARRIET J. DOHENY AND HELEN G. KUEBLER
Franklin High School, Seattle, Wash.

"MATHEMATICS is like a chest of tools: before studying the tools in detail a good workman should know the object of each, when it is used, how it is used, what it is used for."—W. W. Sawyer, Manchester University, England.

The teacher of mathematics who has been trained in the drill and gymnastics of his subject feels the need for assistance in broadening his understanding of the "when," "how," and "what" in the use of mathematics. He knows the subject matter thoroughly but is unacquainted with the details of its use in business, science, and industry.

Professor W. W. Rankin of Duke University, being keenly aware of this situation, has worked out a unique program of teacher training. At a methematics institute each summer he brings leaders from the fields of industry and science to the teachers to tell them what mathematics is used in these fields, how and when it is used. The teachers are introduced to the consumers of the products of their classrooms. Not only do they discover the requirements for various types of work but they see mathematics put to practical use. Then they can enrich their own teaching. They can teach technique by means of applications.

The seventh annual session of the Institute for Teachers of Mathematics was held at Duke University from Aug. 5–15, 1947. Professor Rankin was its director and Miss Veryl Schult, Director of Secondary Mathematics in Washington, D. C., served her fourth term as assistant director. The theme of this year's institute, "Mathematics at Work," was the core around which all of the lectures and study group discussions were built.

* See THE MATHEMATICS TEACHER, May 1947, pages 254–256, for the official program.

One of the first addresses was given by Dr. Harry Soodak, Physicist from Clinton Laboratories at Oak Ridge, Tennessee. He showed that the principles of physics underlying the development of atomic energy are not new, they have been worked out for the last two hundred years and have mathematics as a fundamental tool. If no mathematician had ever lived, the atomic age would still be in the future and would have been delayed by the time required for the physicist to invent the necessary mathematics. Calculus, matrix algebra, differential equations, all have their everyday uses in physics but many situations are expressible in terms of elementary mathematics such as simple algebra.

Mr. R. D. Evans, Mathematician for the Goodyear Tire and Rubber Co. of Akron, Ohio, remarked, "When a better America is built, mathematics will have helped to build it." Mathematics has contributed an important part in the design and development of pneumatic tires. During the last decade the Tire Industry was confronted with projects involving larger and larger tires in an increasing variety of types of operation. Heating to a precise temperature on a precise time schedule must be accomplished with the necessary exactitude. Predetermining the dimensions of tires makes use of elliptic and in some cases hyper-elliptic integrals. Modern industry needs the mathematician, and will continue to need him more and more if our American way of life is to survive, because it will have to depend more and more on technology and on production for survival.

In discussing "Ways and Means of Closer Cooperation Between Industry and Education," R. E. Gilmore, Vice President, of the Sperry Gyroscope Corpora-

tion, showed that our chemists, physicists, physiologists and engineers are taught how to apply their knowledge in terms of social importance. Yet our social sciences impart no reliable skill in making use of their principles in solving problems that arise in the dealings of Man with Man.

Continuing the same theme, Dr. Dwayne Orton, Educational Director of the International Business Machines Corporation, said, "In the task of developing the human values in American industry, education is the greatest ally of labor and management. Education is the process through which human personality is developed. Education must, therefore, be so built into industry as to be the leaven in the loaf, constantly influencing the policies and operating processes of industry in such fashion as to place human values first."

Many valuable suggestions in the making and using of films, instruments, and models were presented by Mr. Roger Zinn of Jam Handy Organization and Miss Frances Burns of Oneida, New York. Audio-visual aids are not to be used as gadgets to amaze but should be used for the enrichment of experience. It was suggested, in fact it was the theme pervading the programs of the institute, that more generous use be made of the community as a laboratory for the teaching of mathematics, that more models, charts, and films be brought into the classroom, that subjects be related to the practical affairs of life, and like Antaeus come back to earth and gain new strength therefrom.

In her talk on "The Role of the Junior High School in the Mathematics Program," Miss Mary Rogers, Chairman of the Research Committee on Junior High School Mathematics, Westfield, N. J., felt that the junior high school, as a connecting link between the elementary school and the senior high school, must assume much of the responsibility for the success of the entire program of mathematical education. The road to mastery in mathematics is a long one and progress thereon must be made continuous by the cooperative effort of teachers from all levels.

Dr. John Curtiss of the Bureau of Standards, Washington, D. C., told of the trend in applied mathematics represented by the development of large scale computing machinery. He gave some typical speed comparisons: To evaluate the determinant of the matrix arising in standard flutter analysis of an aircraft wing by present methods requires 5000 man-hours; by high speed automatic machinery, 1 hour. To solve a partial differential equation arising in explosion theory by present methods, 1200 man-hours; by high speed automatic machinery, $\frac{1}{2}$ hour. To perform a sequence of 100,000 multiplications of pairs of 5 digit numbers and cumulate the sums by present methods, 90 man-hours; by high speed automatic machinery, 5 minutes.

Professor W. D. Reeve, Teachers College, Columbia University reviewed the criticisms of deficiency in arithmetical skills which handicapped service men in the recent war and mentioned some of the steps taken during the war to remedy the situation. He said that although the war is over, the need still remains to increase the mastery and maintenance of arithmetical skills, emphasizing the fact that these skills must be taught directly and not left to incidental treatment. He recommended (1) both mathematics and science should be revised to provide more illustrations and applications; (2) teachers should demand less subject matter but insist on more complete mastery of that used, and cultivate ability to transfer mathematical and scientific learnings to practical situations.

Mathematics in Mechanisms, in Watch Making, in Automatic Packaging Machines, in Ophthmalic Lenses, and Uses of Probability in Manufacturing gave applications of mathematics in various other fields. These topics were discussed by Glenn M. Tracy of Wright Automatic Machinery Co., B. L. Hummel of Hamilton Watch Co., John W. May of Wright Automatic Machinery Co., Dr. Paul Boeder of American Optical Co., and D. K. Briggs of Western Electric Co.

Other lectures were "Engineering Methods of Study" by Professor Vail of Duke University; "Practical Uses of the Limit Concept," Professor G. H. Mumford of North Carolina State College; "The Number System and One to One Correspondence," Professor Dressel of Duke University; "Problem Sources and Solving," Professor Cell of North Carolina State College; "Curves Used in Engineering," Professor Seeley of Duke University.

Seven study groups under the direction of outstanding mathematics leaders covered "Aids in the Study of Geometry"; "Junior High School Mathematics"; "Making and Using Films, Instruments, and Models"; "Enrichment of Mathematics"; "Tests and Measurements"; "Field Work"; "Applications of Mathematics to Science and Engineering."

Those attending the institute were invited to visit the Wright Automatic Machinery Co. of Durham, N. C.; and the Western Electric Company in Burlington, N. C. There were also many delightful social affairs to round out the eleven day program.

Almost two hundred enthusiastically attended the lectures and study groups. No college credit was given for attendance and there were no examinations, but the members learned first hand more than could ever be given in a college course.

Information on previous Duke University institutes may be obtained from THE MATHEMATICS TEACHER for February 1947. Plans are now underway for the 1948 Institute.

EDITORIAL

The Value of Mathematics

IN A recent letter to the Editor, Albert Edward Wiggam, Editorial Director of National Newspaper Service, said:

> It is amazing—truly appalling—how a little remark may influence a boy's or girl's life. When in high school I read the remark of some d—— fool that "mathematics dried up the imagination." From then on through college I dodged all the mathematics I could, although up to that time I was fairly good, and my brother was a star. But I felt I did not wish to lose my imagination!
>
> In later life when I came to study biology, especially genetics, and went on into psychology, both of which are mostly mathematical, I certainly regretted the lost years in learning the fundamentals of mathematics necessary for making a living. It has been a great handicap to me.
>
> When people say to me, "I never could learn mathematics. I just don't have mathematical ability" I tell them "Then you had a mighty poor teacher. Anybody who can come in out of the rain can learn basic mathematics—indeed all the basic learning skills." And they should learn these in the first and second grades—that is, they should learn how to learn. And this depends immensely on the teachers.

How many Americans would give like testimony about the kind of guidance given them in the secondary school regarding mathematics? Here is where the new *Guidance Pamphlet in Mathematics* should be of value today in helping teachers give pupils better advice in regard to the value of mathematics.

At the same time, we all know that some of the mathematics still taught in the schools has little chance of being of value to large groups of pupils who have to study it. Here is a great opportunity for us to improve the organization of mathematics for teaching purposes, and to see to it that an affirmative case can be made out for everything that is included, no matter for whom each course is intended. For some the course may be the sequential subjects, algebra, geometry, and so on. For others it will undoubtedly be general mathematics in the best sense.—W.D.R.

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IN OTHER PERIODICALS

By Nathan Lazar
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The American Mathematical Monthly

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1. Gödel, Kurt, "What Is Cantor's Continuum Problem?" pp. 515–525.
2. Wilks, S. S., "Personnel and Training Problems in Statistics," pp. 525–528.
3. Weaver, C L., "A Simple Analytic Proof of a General x^2 Theorem," pp. 529–533.
4. Williams, O. T., and Browne, D. H., "A Family of Integers and a Theorem of Circles," pp. 534–536.
5. Mathematical Notes, pp. 537–544.
   a. Thébault, Victor, "Theorem on the Trapezoid"
   b. Droussent, Lucien, "On a Theorem of J. Griffith's"
6. Classroom Notes, pp. 540–544.
   a. Robinson, L. V., "Pascal's Triangle and Negative Exponents"
   b. Green, L. C., "Uniform Convergence and Continuity"
   c. Folley, K. W., "Integration by Parts"
   d. Laws, L. S., "A Simplification of the Second Derivative Test"
   e. Heyda, J. F., "Vector Derivation of the Sine and Cosine Laws in Spherical Trigonometry"
7. Elementary Problems and Solutions, pp. 545–549.
8. Advanced Problems and Solutions, pp. 549–552.
9. Recent Publications, pp. 553–557.
10. Club and Allied Activities, pp. 558–561.
11. News and Notices, pp. 561–562.
12. Official Reports and Communications of *The Mathematical Association of America*, pp. 563–572.

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1. Thompson, O. C., "Mathematics from the Viewpoint of a Businessman," pp. 19–25.
2. Hunter, L. L., "Report of the Mathematics Workshop Held in Wichita Last June," pp. 25–27.
3. Betz, William, "Functional Competence in Mathematics," pp. 27–30.
4. McDonald, J., "Geometric Representations of Hyperbolic Functions," pp. 31–32.
5. McKown, John, and Hawkins, Daryl, "Hyperbolic Functions," p. 32.

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October 1947, Vol. 31, No. 296

1. "The Place of Visual Aids in the Teaching of Mathematics," pp. 193–205.
2. Davenport, H., "The Geometry of Numbers," pp. 206–210.
3. Brookes, B. C., "The Incorporation of Statistics into a School Course," pp. 211–218.
4. Gattegno, C., "Mathematics and the Child," pp. 219–223.
5. Goodstein, R. L., "Commutative Involutions," pp. 224–226.
6. Forder, H. G., "The Cross and the Foundations of Euclidean Geometry," pp. 227–233.
7. Mathematical Notes (1977–2001), pp. 234–256.

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1. Tan, Kaidy, "A New Formula for Combinations," p. 762.
2. Lobaugh, Dean, "Girls and Grades: A Significant Factor in Education," pp. 763–774.
3. Kinsella, John J., "A 'Meaning' Theory for Algebra?" pp. 775–780.
4. Grime, Herschel E., "Aptitude and Ability in Elementary Algebra," pp. 781–784.
5. O'Leary, V. C., "Geometric Illustrations in Teaching Algebra," pp. 802–804.
6. Ruchlis, Hyman, "Mathematics Problems from Atomic Science," pp. 807–816.
7. Problem Department, pp. 835–839.
8. Books and Pamphlets Received, pp. 839–840.
9. Book Reviews, pp. 841–846.

NEWS NOTES

NASSAU COUNTY MATHEMATICS TEACHERS ASSOCIATION

The fall meeting of the Nassau County Mathematics Teachers' Association was held in the Oceanside High School, Oceanside, L. I., Wednesday, November 19, 1947. The meeting was opened by the new president, Mr. John J. Clark. He gave a resume of the plans for the Association for the coming year. Interesting speakers and panel discussion are in the offing.

The speaker of the evening was Colonel Hartung of the Academy of Aeronautics, La Guardia Field. His topic was "Education for the Needs of Industry," as they applied particularly to the Aeronautics Field and generally to all Industry. A question period followed.

Refreshments were served and the meeting adjourned at 10:00 P.M.

ALEXANDER K. SOKOL, *Secretary*

The Women's Mathematics Club of Chicago and Vicinity met in Mandel's Tea Room for a luncheon meeting on Dec. 6th, 1947, Mr. Joseph A. Nyberg of Hyde Par, High School spoke on the topic "Ten Minute Tests in Geometry." At their November meeting the club had as their speaker Mr. Allen of La Grange High School who is president of the Men's Mathematics Club who spoke on "Notes on the Teaching of Geometry."

Officers

President, Miss Edith Levin, Englewood High School.

Vice-President, Mrs. Mary Werkman, Parker High School.

Secretary, Miss Marie Brennecke, Washington High School.

Treasurer, Miss Edith Inks, Oak Park High School.

Program Chairman, Miss Bernice L. von Horn, Hyde Park High School.

Joint luncheon and panels of Association of Chairmen of Department of Mathematics and the Association of Teachers of Mathematics of New York City were held at Teachers College on March 29, 1947.

PANEL DISCUSSIONS

Panel I—The Proposed New Tenth Year Course

Chairman: Mr. Joseph B. Orleans, George Washington High School

a. Omissions and Additions. Mr. Henry H. Shanholt, Abraham Lincoln High School.
b. The Unit on Co-ordinate Geometry. Miss Rosemary Tighe, Forest Hills High School.

Panel II—General Mathematics

Chairman: Dr. Eugenie C. Hausle, James Monroe High School.

a. What should be some of the highlight of a course in General Mathematics? Prof. Virgil S. Mallory, State Teachers College, Montclair, N. J.
b. The Problem of introducing the course in the Senior High School. Supt. David Moskowitz, High School Division.
c. General Mathematics at work in the classroom of our school. Mr. Harry Sitomer, New Utrecht High School.

Panel III—Recent Criticisms of the Teaching of Mathematics in High Schools

Chairman: Mr. Max Peters, Long Island City High School.

1. Prof. G. G. Roth, New York University.
2. Mr. Russell Loucks, C.C.N.Y.
3. Mr. Irving Adler, Straubenmuller Textile High School.

Panel IV—The Impact of the Activity Program on Junior High School Mathematics.

Chairman: Miss Dorothy B. Landau, Port Richmond, High School.

a. The Philosophy of the Activity Program as it affects the Junior High School with particular reference to Mathematics.
   1. As the supervisor sees it. Mrs. Irene Taub, Principal, Jr. High School 44, Bronx.
   2. As the classroom teacher sees it. Miss Harriet Coblenz, Jr. High School 44, Bronx.
b. Implications of the Activity Method for the teaching of Junior High School Mathematics. Mrs. Lorraine Addelston, Jr. High School 159, Manhattan.
c. What the Senior High School Mathematics teacher expects of the Junior High School graduate. Mr. Abraham I. Goodman, New Utrecht High School.

Luncheon Program

NOTIONS OF MESSAGE AND ENTROPY

An Address by Professor Nobert Wiener of M.I.T.

The Men's Mathematics Club of Chicago and the Metropolitan Area helds its second meeting on Friday Nov. 21, 1947 at the Central Y.M.C.A. Mr. John F. Schact, Instructor in Mathematics at Bexley Ohio High School was the speaker. His topic was "The Use of Flexible Devices in the Teaching of Geometry." Mr. Schacht has developed a number of such devices and they are now being manufactured by the Welch Scientific Co. of Chicago.

Officers 1947–1948

H. C. TORREYSON, *president,* Lane-Technical High School, Chicago, Illinois.

W. G. HENDERSHOT, *Sec.-Treas.,* Roosevelt High School, Chicago, Illinois.

H. T. DAVIS, *Honorary President,* Northwestern University, Evanston, Illinois.

DAVID RAPPAPORT, *Rec. Sec.,* Lane Technical High School, Chicago, Illinois.

The California Mathematics Council held its Fall Conference on Dec. 19th and 20th, 1947 at the South Pasadena-San Marino Junior High School.

Program

Friday, December 19, 1947

4:00–5:30—Reception in Library
4:30–5:30—Executive Committee Meeting
5:45–7:15—Dinner Meeting—Committee Reports

7:30–9:00—General Session

Topic: "Contribution of Mathematics to Twentieth Century Living"

Speaker: Lionel DeSilva, Executive Secretary, California Teachers Association, Southern Section

Chairman: Reuben R. Palm, Director, Division of Secondary Education, Los Angeles County Schools

Music by South Pasadena-San Marino Junior High School Orchestra, Directed by Charles Mendenhall

Saturday, December 20, 1947

9:15–10:25—Business Meeting—Committee reports and introduction of new officers for 1948

10:30–12:00—General Session

Topic: "The Mathematics Needed to Cope with Twentieth Century Living"

Speaker: Edwin A. Lee, Dean, School of Education, University of California at Los Angeles

12:15– 1:30—Luncheon Meeting

Topic: "Mathematics Needed in Business"

Speaker: Grace S. Stoermer, Past President of the National Association of Bank Women

1:45– 2:30—Group Meetings

Topic: "Specific Teaching Aids in Mathematics"

Leaders: Clela D. Hammond, State Chairman, Secondary and College Committee

Roy DeVerl Willey, State Chairman, Elementary Committee

Richard Wooton, State Chairman, Junior High Committee

2:45– 3:30—General Session—Reports by Committee Chairmen of Group Meetings

3:30– 4:30—Executive Committee Meeting

ADELINE B. NEWCOMB, *President*

MATHEMATICS TEACHERS SCORE SUCCESS

At the eighty-sixth regular meeting of The Association of Mathematics Teachers of New Jersey held on Saturday November 8 in the Hotel Traymore, Atlantic City, the theme was "Good Mathematics Teaching Through Demonstration Lessons." A committee headed by Miss Bertha A. Stanberg of Atlantic City High School arranged the program.

The sixth grade lesson, with Mrs. May J. Kelly teaching her own pupils from Brighton Avenue Elementary School, Atlantic City, demonstrated how children can be taught to multiply decimal fractions without counting to place the decimal point. Never having seen the rule of adding the number of decimal places in the multiplier and multiplicand to find the number for the product, these pupils first estimated their answers and then placed the decimal point to conform with their estimate. Even the slowest seemed to understand the process. The discussion following the lesson was led by Miss Helen McNair of the Belleville schools.

The second lesson, taught by Dr. William S. Tobey of Long Branch High School, showed how fractions might be retaught. The boys and girls in the class were tenth grade pupils from Long Branch who were aware that their IQ's ranged from 65 to 85 and that their computational ability ranged from average to very poor. Dr. Tobey attempted to replace mechanical fractional computation with meaningful methods. The discussion following this lesson was led by Miss Margaret M. Dunn of Bloomfield Junior High School.

The third lesson was an introduction to the theory of complex numbers with a twelfth grade class. Dr. Howard F. Fehr of State Teachers College, Montclair, guided his students to perceive that while a straight line diagram would suffice for all ordinary numbers, a plane was necessary to represent numbers of the type $a+bi$. He also showed the representation of the rho-theta (ρ, θ) system. Samuel Gordon of New Brunswick Senior High School led the discussion.

Following a luncheon at the Hotel Traymore, Professor Robert M. Walter of New Jersey College for Women introduced Professor A. W. Tucker of Princeton University who discussed "The Evolution of Geometrical Concepts." Professor Tucker pointed out that Euclid was the first "non-Euclidean" geometer, since he was dissatisfied with his own "Parallel Postulate" and, that many years later other mathematicians invented geometries and postulated spaces and surfaces hitherto merely dreamed about.

During the morning program, Miss Mary C. Rogers of Roosevelt Junior High School, Westfield, membership chairman, announced that ninety-eight New Jersey High Schools have 100% of their mathematics teachers members in the association, and that twenty schools have all but one teacher belonging. Due to Miss Rogers' untiring efforts, the goal of 1,000 members this year, seems to be in sight.

The third meeting of The Men's Mathematics Club of Chicago and Metropolitan Area was held in Chicago at the Central Y.M.C.A., 19 S. LaSalle St., on Dec. 19 at 6:15 P.M. Dr. George M. Schmeing, Chairman, Department of Chemistry, Loyola University was the guest speaker. His topic was "The Public Relations of the Mathematics Teacher."

The Association of Teachers of Mathematics in New England held its 45th annual meeting at Boston University on Saturday Dec. 19, 1947.

Saturday, December 13, 1947

Morning Session

10:30 A.M. Business Meeting and Election of Officers.

10:45 A.M. to 12:15 P.M.

Mathematics can be Interesting. Mr. Walter H. Carnahan, *Editor in charge of High School mathematics textbooks, D. C. Heath & Co.*

Perspective Triangles. Prof. Edward S. Hammond, *Bowdoin College, Brunswick, Me.*

Afternoon Session

12:30 P.M. Luncheon.

2:00 P.M. Algebra in Geometry Problems. Prof. Albert A. Bennett, *Brown University, Providence, R. I.*

New Officers

Council for 1948

Miss Margaret E. Macdonald, *President*, Durfee High School, Fall River, Mass.; Prof. Elmer B. Mode, *Vice-President*, Boston Uni-

versity, Boston, Mass.; Mr. M. Philbrick Bridgess, *Secretary-Treasurer*, Roxbury Latin School, West Roxbury, Mass.

Term Expires 1948

Miss Ruth B. Eddy, University of Connecticut, Waterbury, Conn.; Mr. Radcliffe Morrill, Headmaster, Wayland High School, Wayland, Mass.

Term Expires 1949

Miss Katherine F. Horrigan, North Quincy High School, Quincy, Mass.; Mr. Sumner C. Cobb, Phillips Academy, Andover, Mass.

Term Expires 1950

Mr. Chas. H. Mergendahl, Newton High School, Newton, Mass.; Mr. Jackson B. Adkins, Philips Exeter Academy, Exeter, N. H.

Professor Edward Kasner of Columbia University spoke to The Society of Friends of Scripta Mathematica and The Yeshiva Institute of Mathematics in the Horace Mann Auditorium of Teachers College, Columbia University on December 22nd, 1947 at 8 P.M. His topic was "Physical Curves."

BOOK REVIEWS

How to Solve It. By G. Polya. Princeton University Press, Princeton, N. J. 1945. vii+204 pages. Price $2.50.

This book by a man who is first an outstanding problem solver himself and only secondly, a pedagogue is in the reviewer's opinion the most concrete and suggestive discussion available of (a) the heuristic method of teaching, (b) the technique of solving problems, (c) the technique of teaching problem solving.

Part I, title "In the Classroom" lists and explains the use and significance of a series of questions to be asked of oneself or of a class in attacking a new problem. These questions involve the ideas subsidiary to the four chief steps in problem solving as conceived by Polya: namely, (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, (4) looking back. Well-chosen concrete examples and a clear exposition of the significance and utility of each question and the corresponding step in the solution of a problem characterize this chapter.

"How to Solve It, A Dialogue" is the title of Part II which in four pages summarizes and reorganizes the procedures of Part I.

These two chapters should be read by every mathematics-teacher-to-be. The book should also be available in high school libraries and recommended to high school and college students. Experienced problem solvers and teachers will find in it many things already known and often used, but even for such people to have these procedures isolated, made explicit, emphasized, and analyzed for their value and implications should be valuable. They will certainly be stimulated to re-evaluate and reorganize their own procedures in the difficult task of teaching problem solving. This book should also lead even experienced teachers to see more clearly to nature and values of the heuristic method of teaching.

Part III, a "Short Dictionary of Heuristic" makes pleasurable and profitable browsing when opened to any page. It contains humor, interesting mathematics, and many profitable suggestions for teaching and problem solving.

Polya repeatedly emphasizes and illustrates several important principles that are frequently slighted in both the preparation and practice of teachers. Among these are the importance of displaying a mathematical motivation for proofs and procedures, of suggesting that students be led to see how they could themselves have evolved an approach to the formulation and proof of a theorem. In this connection Polya frequently notes the importance of intuition, analogy, induction in mathematical thinking and discovery, and he suggests that an understanding of this importance by students would reveal to them more of the aliveness of mathematics and encourage them to more and more effective mental experimentation themselves. Well-known pedagogical principles also acquire new significance when discussed or illustrated by Polya. Such principles are the need for concreteness, the operation of the principle of interval and the action of the subconscious, the desirability of establishing the connections of new ideas with both special cases met earlier and with their generalizations.

In finally recommending the book to all seachers and problem solvers whether in training or in service the authority with which the author may speak should be noted. G. Polya is a Hungarian who studied and taught in Germany, France, and England. He is now settled in this country having come from Zurich where he taught at l'Ecole Polytechnique from 1914 to 1928. He is starred in *American Men of Science* as evidence that his peers regard him as one of this country's leading scientists. His best known works are his *Aufgaben und Lehrsätze der Analysis* written with G. Szego, and *Inequalities* written with G. H. Hardy and J. E. Littlewood. Polya's interest in heuristic has persisted for many years. The present book may be regarded as an expansion of his article "Wie sucht man die Lösung mathematischer Aufgaben?" which

first appeared in *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, 63 (1932) pp. 159–169 and which was then translated for the *Mathematics News Letter* by Professor Norman Anning.—PHILLIP S. JONES.

The Psychology of Invention in the Mathematical Field. By Jacques Hadamard. Princeton University Press, Princeton, N. J. 1945. xi+143 pages. Price $2.00.

This book has a number of things in common with that reviewed above: namely, its publisher, its size, that it has a foreign author, now returned to his native France, who is a distinguished and original mathematician, that this author is summarizing the results of many years of interest in the question of how does one get a new idea.

This book differs from Polya's in that by "invention" a somewhat higher level of originality in the discovery of new facts is implied than by Polya's "problem solving." Hadamard's discussions are general, psychological, and philosophical, in contrast to Polya's concrete practicality. Both Hadamard and Polya reveal aspects of the role of intuition, analogy, and induction in mathematical thinking. Hadamard compares the "sudden illumination" that sometimes produces a really new mathematical invention with poetical and musical inspiration. He relates various anecdotes, citing and amplifying the writings of Poincaré on this topic.

Chapters II and III deal with the unconscious and its relation to discovery. One explanation of the mental machinery of discovery is that "Invention is discernment, choice," that to invent many combinations of ideas must be made and out of them the useful ones must be selected. Many of these combinations may be made and rejected in the unconscious mind. Choices are partially at least guided by esthetic feelings for "mathematical beauty, harmony of numbers and forms, geometric elegance."

Chapters IV and V deal with "The Preparation Stage. Logic and Chance" and "Later Conscious Work." In brief, inspiration does not come unless previously work has been done and the way prepared for inspiration, nor does the work end with the inspiration.

In general, readers will find the book interesting for its anecdotes of the ways of thought of many "inventors" including the author's own introspections and a letter in the appendix from Albert Einstein. The reader may also find here a new interpretation for the words "the poetry of mathematics" and may even add something to his own appreciation of mathematics' beauty and fascination.

For the teacher and teaching of mathematics the book's chief significance lies in the emphasis placed on the role of intuition, insight, or synthesis in the understanding of mathematics. Hadamard (p. 104) quotes Poincaré, "To understand the demonstration of a theorem, is that to examine each syllogism composing it? —For some, yes;—For the majority, no.—They wish to know not merely whether all the syllogisms of a demonstration are correct, but why they link together in this order rather than another." Hadamard goes on to say (p. 105) that frequently in rigorous and clear presentations for beginners nothing remains of the synthesis that gives the "leading thread without which one would be like the blind man who can walk but would never know in what direction to go." He adds, "Those to whom such a synthesis appears 'understand mathematics'." In this day of teaching for meaning and understanding, the implications of these statements should be considered carefully.

Finally then, this book is interesting and worthwhile reading for teachers of mathematics, but it is not in the "must" category of Polya's *How to Solve It.*—PHILLIP S. JONES.

Euclidean Geometry: Its Nature and Its Use. By J. Herbert Blackhurst. Garner Publishing Co., Des Moines, Iowa. 208 pages. $3.00.

This book is written from a metaphysical rather than a scientific point of view.

The first half deals with the nature of Euclidean Geometry. The analysis is an elaboration of the following definition: "The propositions of Euclidean geometry in contrast with pure mathematics, symbolic logic generally, and non-Euclidean geometry, are simple, declarative sentences representing a material content." This definition seems to indicate that the author does not recognize Euclidean Geometry as an abstract mathematical system and that he wishes to consider it only as a branch of applied mathematics (physical geometry). Later on, however, he states: "Known space is Euclidean and can be conceived in no other way." It is clear that here we have a return to the ancient Greek notion of postulates as self-evident truths; the author's development of this idea also indicates his acceptance of Kant's dictum of *a priori* spatial intuition. Thus this conception of the nature of Euclidean Geometry is untenable by either modern pure mathematicians or modern physicists.

The second half of the book is devoted to educational implications. Some interesting ideas on the teaching of reasoning are presented. However, since the educational philosophy is dependent on the preceding mathematical philosophy, there is little that can be reconciled with the teaching of postulational thinking.—FREDERICK W. BORGES